Analysis of Six-legged Walking Robots

Shibendu Shekhar Roy[1*], Ajay Kumar Singh[1], and Dilip Kumar Pratihar[2]
[1] Department of Mechanical Engineering, National Institute of Technology, Durgapur, India
[2] Department of Mechanical Engineering, Indian Institute of Technology, Kharagpur, India

[*] Corresponding author (email: ssroy@nitdgp.ac.in)

Abstract
In the present paper, an attempt has been made to carry out kinematic and dynamic analysis of a six-legged robot. A three-revolute (3R) kinematic chain has been chosen for each leg mechanism in order to mimic the leg structure of an insect. Denavit–Hartenberg (D-H) conventions are used to perform kinematic analysis of the six-legged robot. The direct and inverse kinematic analysis for each leg has been considered in order to develop an overall kinematic model of a six-legged robot, when it follows a straight path. The problems related to trajectory generation of legs have been solved for both the swing and support phases of the robot. It is important to mention that trajectory generation problem during the support phase has been formulated as an optimization problem and solved using the least squared method. Lagrange-Euler formulation has been utilized to determine the joint torques. The developed kinematic and dynamic models have been examined for tripod gait generation of the six-legged robot.

Keywords: Kinematic analysis, Dynamic analysis, Tripod gait, Six-legged robot

1 Introduction

A multi-legged robot possesses a tremendous potential for maneuverability over rough terrain, particularly in comparison to conventional wheeled or tracked mobile robot. It introduces more flexibility and terrain adaptability at the cost of low speed and increased control complexity [1]. In order to develop dynamic model and control algorithm of legged robots, it is important to have good models describing the kinematic behaviour of the complex multi-legged robotic mechanism. The mechanism of a legged robot can be considered as a partially parallel mechanism. Waldron et al. [2] analyzed the kinematics of a hybrid series–parallel manipulation system. Although the work on parallel mechanisms [3] forms a basis for legged-robot kinematic analysis, legged walking robots differ from parallel mechanisms in some important respects. As Lee and Song [4] pointed out, the kinematics of a walking machine is complicated due to its many degrees of freedom. Usually legs of walking machines, during walking are lifted and placed according to a gait, so that the topology of a walking machine mechanism changes. Further, the control problem of a walking machine is significantly more complex than that of a parallel mechanism because a walking machine usually possesses more driven joints than that of a parallel manipulator.

Howard et al. [5] discusses the kinematics of a walking machine using vector and screw algebra. Barreto et al. [6] developed the free-body diagram method for kinematic and dynamic modeling of a six-legged machine. Erden [7] investigated the dynamics of a hexapod walking robot in a level tripod gait based on Newton-Euler formulation. Koo and Yoon [8] obtained a mathematical model for quadruped walking robot to investigate the dynamics after considering all the inertial effects in the system. A dynamic model of walking machine was derived by Lin and Song [9] to study the dynamic stability and energy efficiency during walking. Pfeiffer et al. [10] investigated the dynamics of a stick insect walking on flat terrain. Freeman and Orin [11] developed an efficient dynamic simulation of a quadruped using a decoupled tree-structure approach.

Due to the complexity of a realistic walking robot, it is not an easy task to include the inertial terms in the modelling. The most of the works on walking dynamics were conducted with a simplified model of legs and body. But, in order to have a better understanding of walking, dynamics and other important issues of walking, such as dynamic stability, energy efficiency and online control, kinematic and dynamic models based on a realistic walking robot design are necessary. Here, an attempt has been made to carry out kinematic and dynamic analysis of a real six-legged robot.

2 Kinematics of Three Joint Leg

The kinematic and dynamic analysis of walking robot can be divided into six main parts. Given position, orientation, velocity, acceleration of the trunk body, initial feet positions and gait pattern, calculate the (i) joint displacements based on suitable foot trajectory, (ii) joint velocities and (iii) joint accelerations, (iv) support feet forces, and (vi) torques values of each joint during transfer and support phases.

To derive the kinematic model, the following assumptions are made:

(a) The robot moves forward in a straight path on flat surface with alternating tripod gait.

(b) The trunk body is held at a constant height and parallel to the ground plane during locomotion.

(c) The center of gravity of the trunk body is assumed to be at the geometric center of the body.

Fig. 1 shows a 3-D model of a six-legged walking robot considered in the present study. It consists of a trunk of rectangular cross-section and six legs, which are similar and systematically distributed around the trunk body on two sides. Each leg has three degrees of freedom and is composed of three links connected by the three rotary joints. The Denavit-Hartenberg (D-H) notations [12] have been used in kinematic modeling of each leg (refer to Fig. 2).

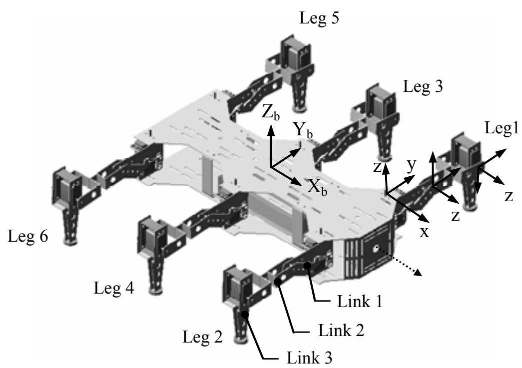

Fig. 1 CAD model of six-legged robot

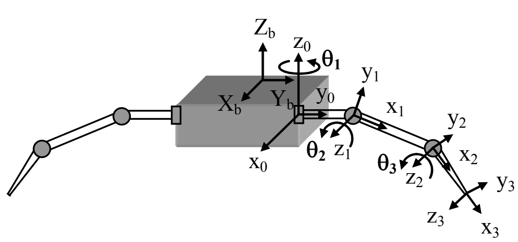

Fig. 2 D-H representation of link frame

Tables 1 and 2 show four D-H parameters, namely link length (a_i), link twist (α_i), joint distance (d_i), and joint angle (θ_i), required to completely describe the three joint legs.

Table 1: D-H parameters for left legs

Link	a_i	α_i	d_i	θ_i
1	L_1	90°	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

Table 2: D-H parameters for right legs

Link	a_i	α_i	d_i	θ_i
1	L_1	-90°	0	θ_1
2	L_2	0	0	θ_2
3	L_3	0	0	θ_3

The links' homogeneous transformation matrices have been presented as given below.

$$
{}^{0}\mathbf{T}_1 = \begin{bmatrix} c\theta_1 & 0 & s\theta_1 & L_1 c\theta_1 \\ s\theta_1 & 0 & -c\theta_1 & L_1 s\theta_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{1}\mathbf{T}_2 = \begin{bmatrix} c\theta_2 & -s\theta_2 & 0 & L_2 c\theta_2 \\ s\theta_2 & c\theta_2 & 0 & L_2 s\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$
{}^{2}\mathbf{T}_3 = \begin{bmatrix} c\theta_3 & -s\theta_3 & 0 & L_3 c\theta_3 \\ s\theta_3 & c\theta_3 & 0 & L_3 s\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The resulting transformation matrix between foot tip reference frame {3} and leg or hip reference frame {0} is given as:

$$
{}^{0}\mathbf{T}_3 = {}^{0}\mathbf{T}_1\, {}^{1}\mathbf{T}_2\, {}^{2}\mathbf{T}_3 \tag{1}
$$

$$
{}^{0}\mathbf{T}_3 = \begin{bmatrix} c\theta_1 c(\theta_2+\theta_3) & -c\theta_1 s(\theta_2+\theta_3) & s\theta_1 & \left(L_1+L_2 c\theta_2+L_3 c(\theta_2+\theta_3)\right)c\theta_1 \\ s\theta_1 c(\theta_2+\theta_3) & -s\theta_1 s(\theta_2+\theta_3) & -c\theta_1 & \left(L_1+L_2 c\theta_2+L_3 c(\theta_2+\theta_3)\right)s\theta_1 \\ s(\theta_2+\theta_3) & c(\theta_2+\theta_3) & 0 & L_2 s\theta_2+L_3 s(\theta_2+\theta_3) \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The six legs and the trunk body must be integrated to solve the kinematic problem of the robot. Consider the body attached reference frame is located at the geometric center of the trunk body. Leg 'i' coordinates in body reference frame are obtained using transformation matrix as given below.

$$
{}^{b}\mathbf{T}_{0,i} = \begin{bmatrix} 1 & 0 & 0 & L_{xi} \\ 0 & 1 & 0 & L_{yi} \\ 0 & 0 & 1 & L_z \\ 0 & 0 & 0 & 1 \end{bmatrix}
$$

The x, y and z coordinates of the foot tip point with respect to leg reference frame {0} can be determined for given the joint variables: θ_1, θ_2 and θ_3. The position of the foot is given by the following expressions:

$$[L_1+L_2 \cos\theta_2+L_3 \cos(\theta_2+\theta_3)] \cos\theta_1 = p_x \tag{2}$$

$$[L_1+L_2 \cos\theta_2+L_3 \cos(\theta_2+\theta_3)] \sin\theta_1 = p_y \tag{3}$$

$$L_2 \sin\theta_2+L_3 \sin(\theta_2+\theta_3) = p_z \tag{4}$$

By solving equations (2), (3) and (4), the joint angles : θ_1, θ_2 and θ_3 have been determined as given below.

$$\theta_1 = \operatorname{atan2}(p_y, p_x) \tag{5}$$

$$\theta_2 = \operatorname{atan2}\left(c, \pm\sqrt{a^2+b^2-c^2}\right) - \operatorname{atan2}(a,b) \tag{6}$$

where $a = 2L_2\left(\sqrt{p_x^2+p_y^2} - L_1\right)$;

$b = 2p_zL_2$; $c = \left[\left(\sqrt{p_x^2+p_y^2} - L_1\right)^2 + p_z^2 + L_2^2 - L_3^2\right]$.

$$\theta_3 = \cos^{-1}\left[\frac{\left(\sqrt{p_x^2+p_y^2} - L_1\right)^2 + p_z^2 - L_2^2 - L_3^2}{2L_2L_3}\right] \tag{7}$$

3 Foot Trajectory Planning

The robot is assumed to describe a continuous alternating tripod gait (refer to Fig. 3) that consists of two main phases. In the first phase, legs: 1, 4, and 5 are in support and moving backwards at a specified trapezoidal velocity profile, while legs: 2, 3, and 6 are in their swing phase, moving forward to their next footholds. Each supporting foot tip follows a straight-line trajectory on the ground parallel to the trajectory of other supporting feet.

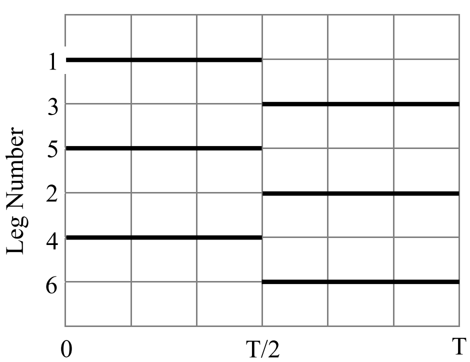

Fig. 3 Gait Diagram (duty factor=0.5)

3.1 Swing Foot Trajectory

To ensure a smooth functioning, each joint trajectory of swing legs is assumed to follow a polynomial of fifth degree in time (t). If θ_j is the angle of j[th] joint of a swing leg, fifth order polynomial can be expressed as follows:

$$\theta_j = a_{j0}+a_{j1}t+a_{j2}t^2+a_{j3}t^3+a_{j4}t^4+a_{j5}t^5, \tag{8}$$

where a_{j0}, a_{j1}, a_{j2}, a_{j3}, a_{j4}, and a_{j5} are coefficients, whose values are determined using a set of boundary conditions defined over the swing phase and j=1, 2, 3 joints. θ_j is considered to be positive in counterclockwise direction. The boundary conditions of angular displacement and angular velocity at initial, middle and final points of the trajectory are applied to find the six coefficients for each joint as shown in Table 3.

Table-3: Coefficient values of joint trajectory

Joint no. (j)	Coefficient values					
	a_{j0}	a_{j1}	a_{j2}	a_{j3}	a_{j4}	a_{j5}
1	110	0	-2.667	-11.259	5.296	-0.790
2	-20.8	0	3.004	8.663	-6.777	1.185
3	-61.8	0	0.116	0.367	-0.283	0.0494

3.2 Support Foot Trajectory

Fig. 4 displays the trapezoidal velocity profile of center of mass of the trunk body for each half cycle. The cycle time and maximum velocity of trunk body are assumed to be equal to 6 sec and 0.056 m/sec, respectively.

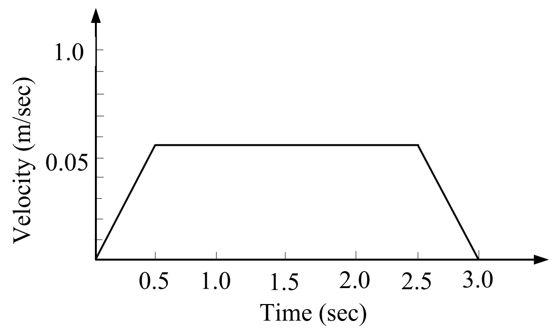

Fig. 4 Velocity profile of the trunk body of the robot

For smooth variation of the joint angles, their trajectories are assumed to follow the fifth order polynomial as shown below.

$$\theta_j = c_{j0}+c_{j1}t+c_{j2}t^2+c_{j3}t^3+c_{j4}t^4+c_{j5}t^5, \tag{9}$$

where c_{j0}, c_{j1}, c_{j2}, c_{j3}, c_{j4}, and c_{j5} are the coefficients. It is to be noted that the half cycle time (3 sec) has been equally divided into thirty intervals. The least squared method has been used to find six coefficients from thirty known values of θ_j. The optimized coefficients are summarized in Table 4.

Table-4: Optimized coefficient values of joint trajectory

Joint no. (j)	Coefficient values						
	c_{j0}	c_{j1}	c_{j2}	c_{j3}	c_{j4}	c_{j5}	c_{j6}
1	70.16	-2.66	28.82	-20.8	7.17	-0.95	0.0
2	-20.80	-0.24	3.19	-4.79	3.06	-0.91	0.10
3	-62.03	3.50	-33.19	39.92	-22.1	6.19	-0.69

4 Dynamics of Six-legged Robot

For deriving the dynamic equations and finding joint torques' variations over the locomotion cycle, Lagrange-Euler formulation has been used. The direct application of Lagrangian dynamics formulation together with Denavit-Hartenberg's link coordinate representation results in a convenient, compact, systematic algorithmic description of the equations of motion. A systematic derivation of Lagrange-Euler equations yields a dynamic expression that can be written in the vector-matrix form as given below.

$$\boldsymbol{\tau} = \mathbf{M}(\boldsymbol{\theta})\ddot{\boldsymbol{\theta}} + \mathbf{H}(\boldsymbol{\theta}, \dot{\boldsymbol{\theta}}) + \mathbf{G}(\boldsymbol{\theta}) + \mathbf{J}^{\mathrm{T}}\mathbf{F}, \qquad (10)$$

where $\mathbf{M}(\theta)$ is the 3×3 inertia matrix of the leg, $\mathbf{H}$ is a 3×1 vector of centrifugal and Coriolis terms, $\mathbf{G}(\theta)$ is a 3×1 vector of gravity terms, $\boldsymbol{\tau}$ is the 3×1 vector of joint torques and $\mathbf{F}$ is the 3×1 vector of ground contact forces. During the leg's swing phase, there is no foot-terrain interaction, and $\mathbf{F}$ becomes equal to zero. However, during the support phase, ground contact exists and equation (10) becomes undetermined. For computing foot-force distributions, the following assumptions are made:
(i) The ground legs are assumed to be supporting the trunk body without any slippage on their tip points.
(ii) The contacts of the tip of the feet with ground can be modeled as hard point contacts with friction, which indicates that the interaction between the tip of the leg and ground is limited to three components of force: one normal and two tangential to the surface.

Let us assume that $\mathbf{F}_{pqr}=[\mathbf{F}_p, \mathbf{F}_q, \mathbf{F}_r]^T$ is the foot-force vector, when the legs: p, q and r are in support phase, where $\mathbf{F}_i=[f_{ix}, f_{iy}, f_{iz}]^T$ is the ground-reaction force on foot i, where i=p, q, r. In the first phase of tripod gait, p, q and r are the legs: 1, 4, and 5, respectively, and during the next phase, the legs: 2, 3, and 6 will be in the support phase. The wrench $\mathbf{W}=[F_x, F_y, F_z, M_x, M_y, M_z]^T$ contains the forces (F_x, F_y, F_z) and moments (M_x, M_y, M_z) acting on the robot's center of gravity and represents the robot's payload, including the effect of surface gradient, any externally applied forces and inertial effects of the robot's body. However, the inertial effects of the legs have been neglected to simplify the study. Under these conditions, six equilibrium equations that balance forces and moments, when three legs, namely p, q, and r are in their support phase, can be obtained as follows:

$$\sum_{i=p,q,r} f_{ix} + F_x = 0$$

$$\sum_{i=p,q,r} f_{iy} + F_y = 0$$

$$\sum_{i=p,q,r} f_{iz} + F_z = 0$$

$$\sum_{i=p,q,r} y_i f_{iz} - \sum_{i=p,q,r} z_i f_{iy} + y_c F_z - z_c F_y + M_x = 0$$

$$\sum_{i=p,q,r} z_i f_{ix} - \sum_{i=p,q,r} x_i f_{iz} + z_c F_x - x_c F_z + M_y = 0$$

$$\sum_{i=p,q,r} x_i f_{iy} - \sum_{i=p,q,r} y_i f_{ix} + x_c F_y - y_c F_x + M_z = 0$$

These equations are normally written in a matrix form as follows:

$$\mathbf{A}_{pqr}.\mathbf{F}_{pqr} = (-\,\mathbf{B}.\mathbf{W}) \qquad (11)$$

where

$$\mathbf{A}_{pqr} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{I}_3 & \mathbf{I}_3 \\ \mathbf{R}_p & \mathbf{R}_q & \mathbf{R}_r \end{bmatrix}; \text{ and } \mathbf{B} = \begin{bmatrix} \mathbf{I}_3 & \mathbf{0}_3 \\ \mathbf{R}_c & \mathbf{I}_3 \end{bmatrix}$$

$\mathbf{I}_3$ is the (3×3) identity matrix, $\mathbf{0}_3$ is the (3×3) null matrix and $\mathbf{R}_i$ is the (3×3) skew symmetric matrix of vector $[x_i, y_i, z_i]^T$.

$$\mathbf{R}_i = \begin{bmatrix} 0 & -z_i & y_i \\ z_i & 0 & -x_i \\ -y_i & x_i & 0 \end{bmatrix} \text{ and } \mathbf{I}_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

This matrix defines the position of tip of a foot i (i=p, q, r) or that of center of gravity (i=c) with respect to body reference frame. The coordinates of i^{th} foot-ground contact point with respect to body reference frame, located at the body's geometric center, are denoted by (x_i, y_i, z_i).

With the known feet positions, the feet forces during a whole locomotion cycle can be computed using equation (11), which is indeterminate, because it consists of six equations but there are nine unknowns. The solution of equation (11) has been obtained using the least squared method.

5 Simulation Results

In this section, simulation results of the above mathematical model have been discussed in detail. Table 5 shows the physical parameters of each leg of the six-legged robot used in computer simulations. The leg stroke of the tripod gait and body height are assumed to be equal to 0.14 m and 0.13 m, respectively.

Table- 5: Physical parameters of each leg

Link parameters		Link 1	Link 2	Link 3
Mass (kg)	m	0.152	0.04	0.106
Length (10^{-3} m)	L	85	115	100
Moment of Inertia (10^{-4} kg-m^2)	I_x	1.00	0.23	0.22
	I_y	8.28	3.07	10.00
	I_z	9.09	2.91	10.01

Fig. 5 shows the distributions of foot reaction forces of legs: 1, 4 and 6 during their support phase over half locomotion cycle. It is to be noted that similar distributions of foot reaction forces of legs: 2, 3 and 5 during their support phase over half locomotion cycle have been obtained. Moreover, the front and rear legs complement each other in force, such that sum of the vertical forces of all the ground legs at any given instant of time becomes equal to the weight of the robot. It has been observed that the middle legs are subjected to maximum force of up to 19.7 N, while the maximum force acting at corner legs are found to be equal to 15.9 N. It has happened so, due to the fact that the foot force depends on that leg's foot position relative to center of mass of the trunk.

Joint torques are comprised of three components, namely inertial term (M-term), centrifugal and Coriolis term (H-term) and gravity term (G-term).

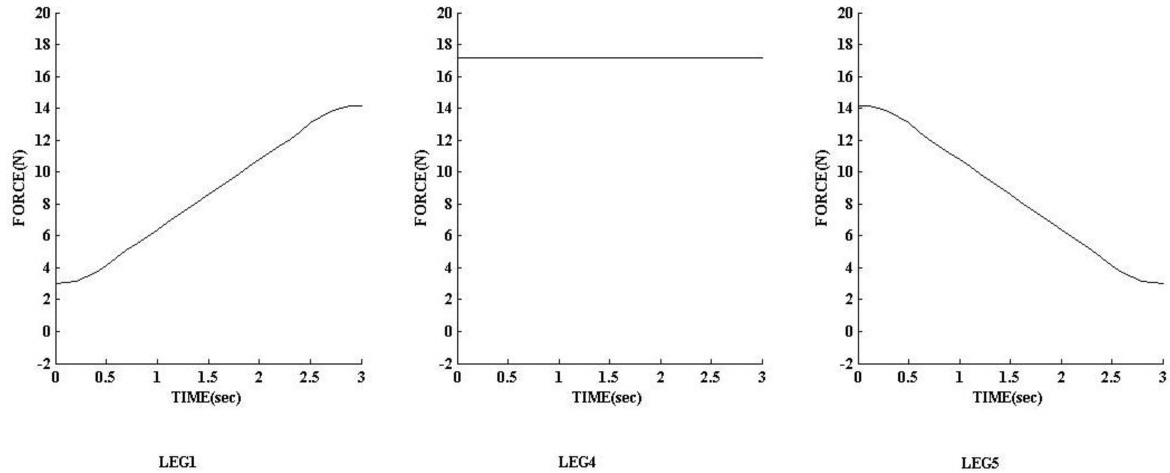

Fig. 5: Foot reaction forces for half cycle (when legs 1, 4 and 5 are on ground)

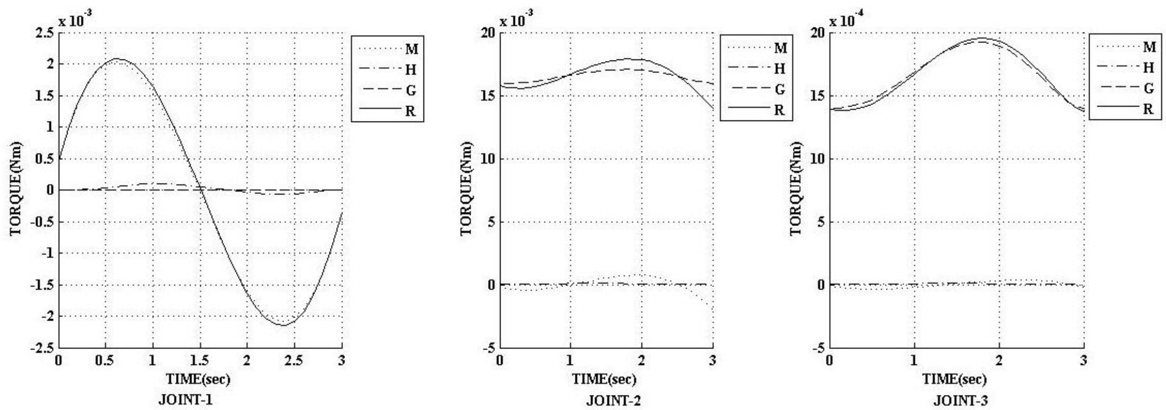

Fig. 6: Contribution of M, H and G terms on torques at joint 1, 2 and 3 during swing phase of left side legs

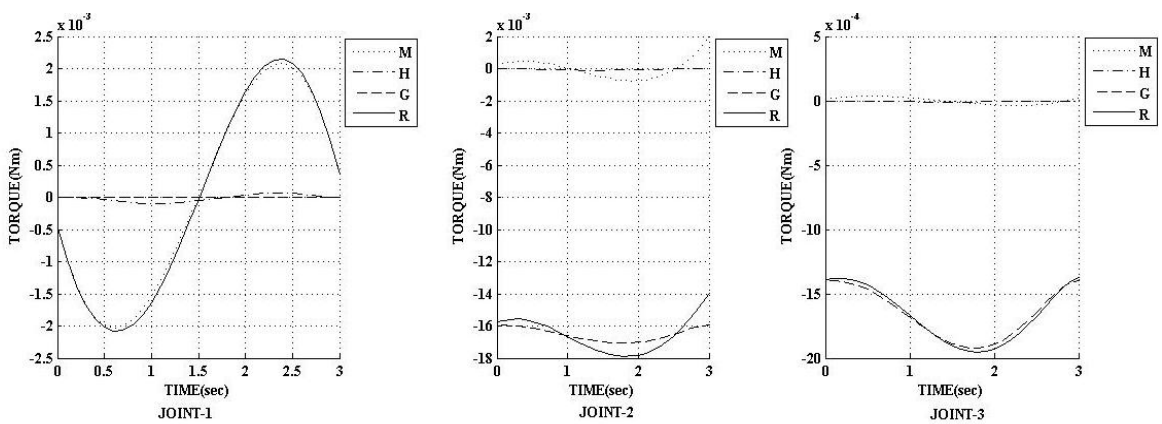

Fig. 7: Contribution of M, H and G terms on torques at joint 1, 2 and 3 during swing phase of right side legs

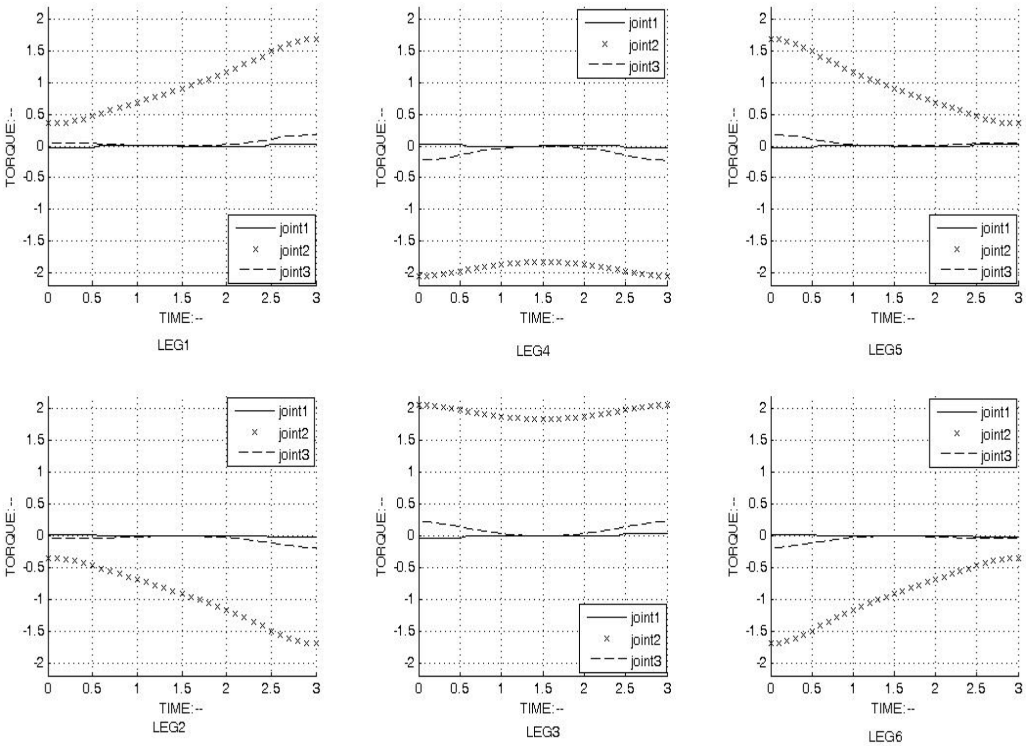

Fig. 8: Variations of joint torques of each leg during their support phase

Figs. 6 and 7 show the contributions of inertia, centrifugal/Coriolis and gravity terms on torque of joints: 1, 2 and 3 during swing phase of left side legs and right side legs, respectively. It is important to note that the gravity has only negligible effect on torque of joint 1, on which the inertia has significant contribution. Torques of joints: 2 and 3 are mainly dependent on the gravity, and the effect of inertia and centrifugal/Coriolis terms are found to be negligible. Fig. 8 represents the variations of joint torques in each joint of the legs during their support phase. It is interesting to note that for a particular ground leg, the maximum torque generated at joint 2 is more compared to that at other two joints. The torque values of joint 2 vary in the range of 1.830 Nm to 2.055 Nm for the middle legs and those for other legs are seen to lie in the range of 0.35 Nm to 1.69 Nm. It is also interesting to note that the maximum torque required at joints: 1 and 3 of all the legs is found to be equal to 0.22 Nm. Thus, the maximum torque required at joint 2 is seen to be about 8 to 9 times of that at other joints (namely joints: 1 and 3). Moreover, joint torques of the legs during the support phase (refer to Fig. 8) have come out to be much more than those during the swing phase (as shown in Figs. 6 and 7), as expected.

6 Conclusions

Both the kinematic as well as dynamic analyzes of a six-legged robot have been carried out in the present study. The direct and inverse kinematic analysis for each leg has been conducted in order to develop the overall kinematic model of a six-legged robot. The problems related to trajectory generation of legs have been solved for both the swing and support phases of the robot. It is important to mention that trajectory planning problem during the support phase has been solved using the least squared method. An attempt has been made in present study to obtain optimal distributions of feet forces. It has been observed that the middle legs are subjected to more force than corner legs. Joint torques have been calculated using Lagrange-Euler formulation of the rigid multi-body system. The developed kinematic and dynamic models have been examined for tripod gait generation of the six-legged robot. This work can be extended to tackle the problems related to tetrapod and non-periodic gait of the walking robot.

Acknowledgment

The first author acknowledges all support from Department of Mechanical Engineering, NIT, Durgapur.

References

[1] S.M. Song and K.J.Waldron, *Machines That Walk: The Adaptive Suspension Vehicle*, The MIT Press, Cambridge, Massachusetts, 1989.

[2] K.J. Waldron et al., "Kinematics of a hybrid series-parallel manipulation system," *Transaction of ASME Journal of Dynamics System, Measurement and Control,* Vol.111, 1989, pp. 211-221.

[3] NS.J. Zhang et al., "Walking machine design based on the mechanics of the Stewart platform", *Proc. of the ASME ESDA Conference, 64-8.3, 849-855 London, 1994.*

[4] J.K. Lee and S.M. Song, "A study of instantaneous kinematics of walking machines," *International Journal of Robotics and Automation,* Vol. 5 (3), 1990, pp. 131-138.

[5] D. Howard et al, "Kinematic analysis of a walking machine," *Mathematics and Computers in Simulation,* Vol. 41, 1996, pp. 525-538.

[6] J.P. Barreto, A.Trigo, P. Menezes, J. Dias, and A.T.de Almeida, "FBD-The free body diagram method. Kinematic and dynamic modeling of a six leg robot", *IEEE Int. Conf. on Robotics and Automation*, pp. 423-428, 1998.

[7] M.S. Erden, and K. Leblebicioglu, "Torque distribution in a six-legged robot", *IEEE Trans. On Robotics*, vol. 23(1), pp. 179-186, 2007.

[8] T.W. Koo, and Y.S. Yoon, "Dynamic instant gait stability measure for quadruped walking", *Robotica*, vol. 17, pp. 59-70, 1999.

[9] B.S. Lin, and S.M. Song, "Dynamic modeling, stability and energy efficiency of a quadrupedal walking machine", *IEEE Int. Conf. on Robotics and Automation*, pp. 367-373, 1993.

[10] F. Pfeiffer, H.J. Weidemann, and P. Danowski, "Dynamics of walking stick insect", *Proc. IEEE Int. Conf. on Robotics and Automations*, vol. 2, pp. 1458-1463, May 1987.

[11] S. Freeman and E. Orin, "Efficient dynamic simulation of a quadruped using a decoupled tree-structure approach", *The International Journal of Robotics Research*, Vol. 10(6), 1991, pp. 619-627.

[12] J. Denavit and R.S. Hartenberg, "A kinematic notation for lower-pair mechanisms based on matrices", *ASME Journal of Applied Mechanics*, Vol. 77, 1955, pp. 215-221.