Stability Analysis of a Two-wheeler during Curve Negotiation under Braking

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Abstract

The two-wheelers show interesting dynamic characteristics. They are statically unstable. But the roll instability disappears as the forward speed increases. Here a simplified model is considered to analyze the effects of forward speed and braking force on the roll instability during cornering of a two-wheeler. Variations of bike parameters are also studied in the relevance of roll stability. The present work helps to understand some important concepts about a two-wheeler negotiating a turn under applied braking force.

Keywords: Cornering, Braking, Roll-stability, Tire properties.

1 Introduction

Steering a two-wheeler involves a complex interaction between centrifugal force, gravitational force and the torque applied to the handle bar together with the geometry of the two-wheeler and rider. Leaning the two-wheeler into the turn and maintaining an appropriate forward speed allows gravitational forces to balance the centrifugal forces, leading to a controlled and stable turn.

One method of establishing the proper lean is counter-steering in which handlebar is turned counter to the desired turn and thus developing a centrifugal torque that leans the two-wheeler appropriately. Alternately the required lean can be generated by throwing the rider's hips in the direction counter to the turn.

The situation becomes even more complicated when a sudden braking force is applied. During braking, the braking forces, the load distribution, changes in the cornering stiffness and the camber stiffness and lateral adherence mainly affects the lateral stability of the two-wheeler. The major problem in applying brake during curve negotiation is the tendency of tire skidding. The stability analysis of the two-wheeler provides important information about its handling capabilities and riding safety.

A number of important studies have been reported on vehicle dynamics and control to optimize the performance of ground vehicle with regard to stability and road handling. Simple models of bicycle dynamics are given in the text book [1]. K. J. Åstrom, R. E. Klein and A. Lennartsson have treated the bicycles from the perspective of control [2]. Models of different complexity are presented and models that capture essential behavior such as self stabilization and demonstrate the difficulties with rear wheel steering are presented. R. S. Sharp and D. J. N. Limebeer discussed about the confirmation and extension of motorcycle model using automated model building platform, AutoSim [3]. V. Cossalter R. Lot and F. Maggio have presented a study on the braking style and its implication on motorcycle stability [5]. The ideal braking strategy and braking maneuvers are analyzed using a detailed multi-body model. D. J. N. Limebeer, R. S. Sharp discussed about different bicycle and motorcycle models for stability and control [6]. David J. N. Limebeer and Amrit Sharma studied the dynamics of the accelerating bicycle under straight-running and cornering conditions [8].

2 Two-wheeler model and Equations

2.1 Two-wheeler model for curve negotiation without braking

For the purpose of the analysis a simple two-wheeler is considered as shown in fig.1. The coordinates used to analyze the two-wheeler is shown in fig.2 (a). The inertial system is with the axes uvw and origin O. The system uvw is fixed to inertial frame and the system xyz has its origin at contact point of rear wheel with the xy plane. The x-axis is in the direction of the contact line of the rear wheel with xy plane and z- axis is vertical and y-axis is perpendicular to x and positive on the left side of the two-wheeler. The roll angle Φ of rear frame is positive when leaning to the right and the steer angle δ is positive for steering left. The angle between axes x and u, is ψ indicating the orientation of rear wheel plane.

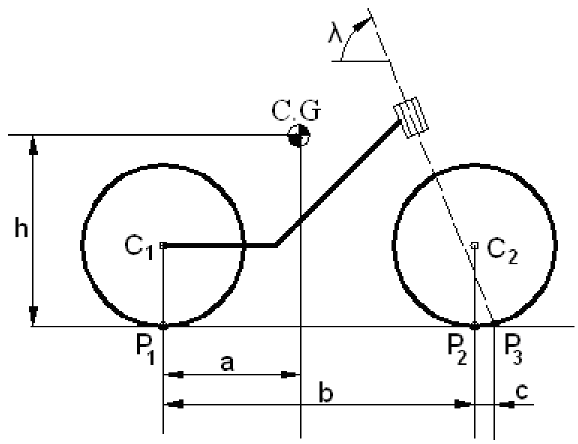

Fig. 1: Parameters defining a simple two-wheeler.

Due to tilt in steer axis by an angle λ, the effective front angle is [2]

$$\delta_f = \delta \sin\lambda \tag{1}$$

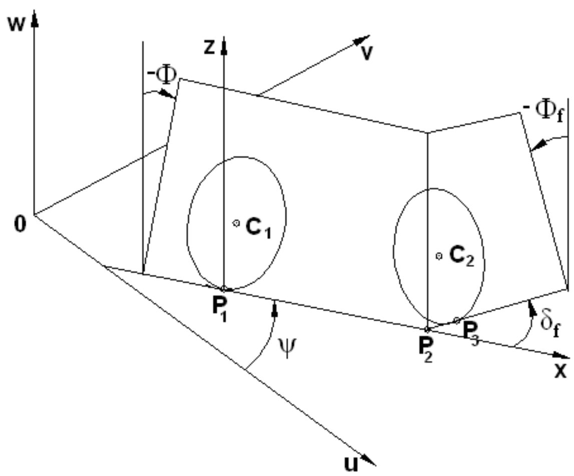

Fig. 2 (a): Coordinate system.

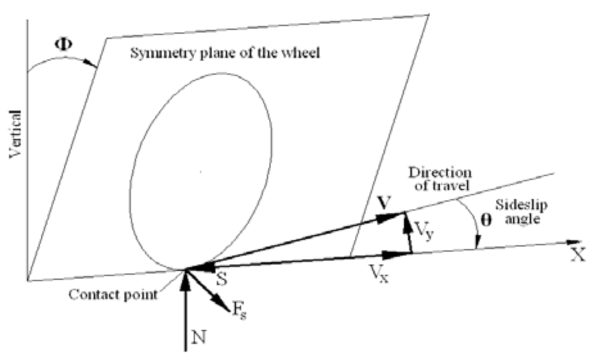

Fig. 2 (b): Tyre forces and sideslip angle.

The effective front fork roll angle is given by

$$\phi_f = \phi - \delta\cos\lambda \tag{2}$$

Normal reaction force on front wheel excluding dynamic and centrifugal effect

$$N_f = \frac{amg}{b} \tag{3}$$

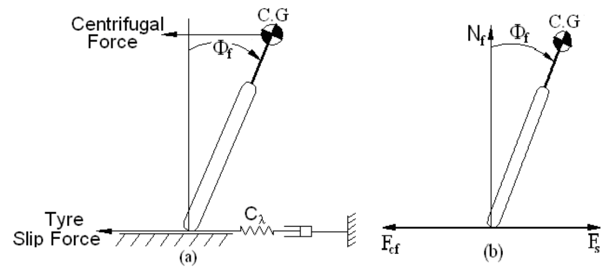

Fig. 3: (a) Tire model, (b) Reaction forces on front wheel.

Effective centrifugal force on front wheel

$$F_{cf} = \frac{amv^2}{b \times r}$$

$$F_{cf} = \frac{amv^2}{b^2}\delta_f \tag{4}$$

$$F_{cf} = \frac{amv^2 \sin\lambda}{b^2}\delta$$

Lateral force resisting sideslip [5]

$$F_s = C_\lambda \times \theta + C_\phi \times \phi_f \tag{5}$$

The static torque balance equation

$$T - [F_{cf} + (C_\lambda \times \theta + C_\phi \times \phi_f)]c \times \sin\lambda = 0 \tag{6}$$

Substituting equations (1) - (5) in equation (6)

$$\delta = \frac{1}{K_\lambda c \sin\lambda}T - \frac{C_\phi}{K_\lambda}\phi - \frac{C_\lambda}{K_\lambda}\theta \tag{7}$$

Angular momentum balance for the two-wheeler frame [2]

$$J\ddot{\phi} - mgh\phi = \frac{Dv\sin\lambda}{b}\dot{\delta} + \frac{m(v^2h - acg)\sin\lambda}{b}\delta \tag{8}$$

Substituting δ from equation (7) in main frame equation (8)

$$J\ddot{\phi} + \frac{DvC_\phi \sin\lambda}{bK_\lambda}\dot{\phi} + [K_v C_\phi \sin\lambda - mgh]\,\phi = \frac{Dv}{bcK_\lambda}\dot{T} + \frac{K_v}{c}T - [K_v C_\lambda \sin\lambda]\theta \tag{9}$$

Where,

$$K_\lambda = \frac{amv^2 \sin\lambda - b^2 C_\phi \cos\lambda}{b^2}$$

$$K_v = \frac{m(v^2 - acg)}{bK_\lambda}$$

$$D = mah$$

$$J = mh^2$$

The above equation reveals that torque at the handle bar has a great influence on roll angle as well as stability of the two-wheeler.

2.2 Two-wheeler model for curve negotiation with braking

For stability analysis of the two-wheeler during cornering with braking the model considered is shown in fig.3. The key purpose of the model is to discuss the balancing problem for two-wheeler during cornering under braking. The distribution of forces during braking is shown in fig.4.

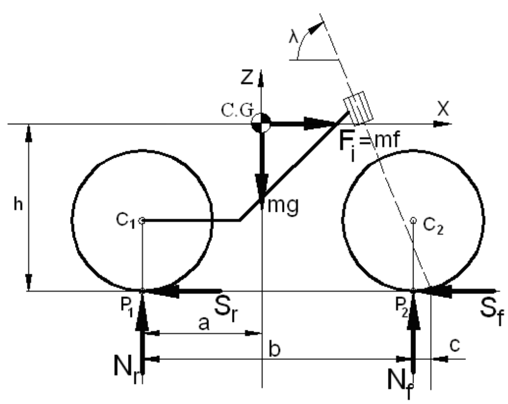

Fig. 4: Forces acting on the two-wheeler during braking.

Vertical reaction force due to braking on front and rear wheel

$$N_{fb} = \frac{amg}{b} + \frac{mfh}{b}$$
$$N_{rb} = \frac{(b-a)mg}{b} - \frac{mfh}{b} \tag{10}$$

Substituting δ in main frame equation (8) from equation (7)

$$J\ddot{\phi} + \frac{DvC_\phi \sin\lambda}{bK_\lambda}\dot{\phi} + \left[K_1 C_\phi - mgh\right]\times\phi = \frac{Dv}{bcK_\lambda}\dot{T} + \frac{K_1}{c\sin\lambda}T - [K_1 C_\lambda]\theta \tag{11}$$

Where,

$$K_1 = \frac{m\sin\lambda}{bK_\lambda}\left[v^2 h - acg - \frac{2Dav^2 \sin\lambda}{b^2 K_\lambda}\right]$$

$$f = \frac{F_i}{m}$$

Equation (11) is derived considering deceleration due to braking and therefore, velocity as a variable with time. Breaking force largely affects the stability of the two-wheeler which can be controlled suitably applying torque on the handle bar. Thus equations (9) and (11) provide qualitative explanation for stabilization.

3 Simulation

Simulation is carried out for a motor cycle using equations (9), (10) and (11) to study the response of roll angle as the two-wheeler negotiates a curve, both at constant speed as well as under uniform braking using a program written in **C** language. The parameters of the two-wheeler used for this simulation are shown in table-1. The flow chart for the programming is as follows:

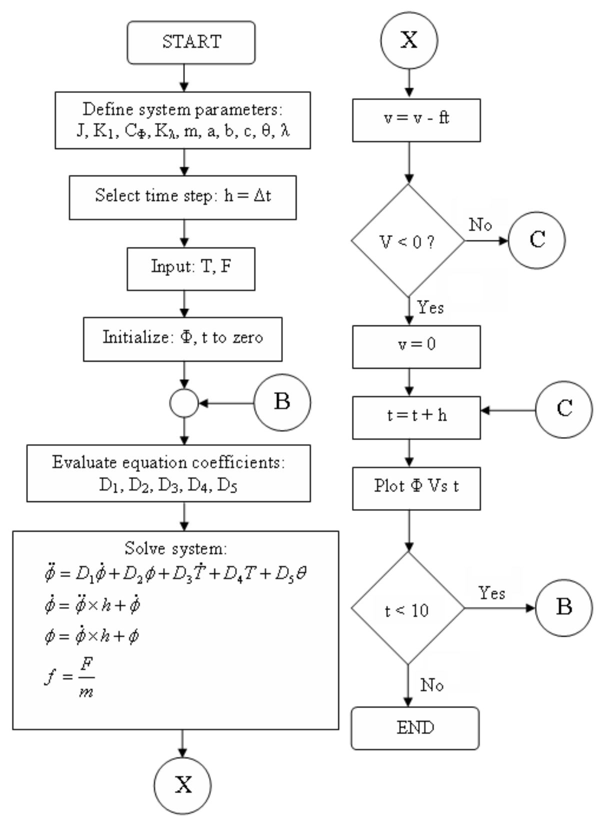

Fig. 5: Flow chart for simulation

Table-1: Two-wheeler parameters

Two-wheeler Parameters:	Value	Unit
Mass (m)	200	kg
Length of wheel base (b)	1.54	m
Position of C.G in plane containing rear wheel (h)	0.6	m
Position of C.G along x-axis from P_1 (a)	0.513	m
Head angle (λ)	70	Deg.
Trail (c)	0.117	m
Sideslip angle (θ)	2	Deg.

The two-wheeler shows unstable roll behaviour unless the rider applies a suitable torque at the handle-bar. Rider is assumed to behave like a PID controller by applying a steer torque proportional to bicycle lean [9]. The handle bar torque

$$T = -K_p(\phi_d - \phi) + K_d\dot{\phi} + K_i\int_0^t (\phi_d - \phi)dt$$

Where, K_p, K_d and K_i are controller gains.

Tire parameters C_λ and C_Φ changes with vertical load and braking force [5]. The response of roll angle is studied during cornering with a desired lean (ϕ_d) of 0.5 radian (approx. 30^0) at different running speed of the two-wheeler — first, without braking and then with several braking forces. The effect of braking is plotted (fig.6) at a running speed of 20 km/hr.

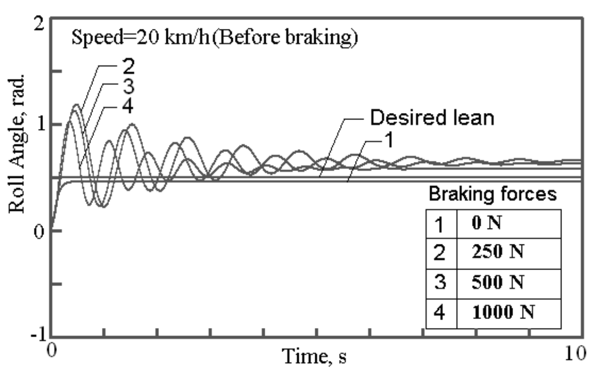

Fig. 6: Roll response of the two-wheeler as it negotiates a curve with and without braking forces.

4 Results and Discussion

Stability analysis for the two-wheeler is carried out under various running speeds without any braking force as well as applying different magnitudes of braking forces. The left hand side of the system equations (9) and (11) and the root locus plot for the corresponding characteristic equation (fig.7) reveals that the system is unstable with the characteristic roots falling at the right hand side of the complex plane. The inherently unstable two-wheeler can only be made stable with an appropriate steering torque provided at the handle bar by the rider. It is also seen that such instability of the two-wheeler gradually decreases with the increase in speed.

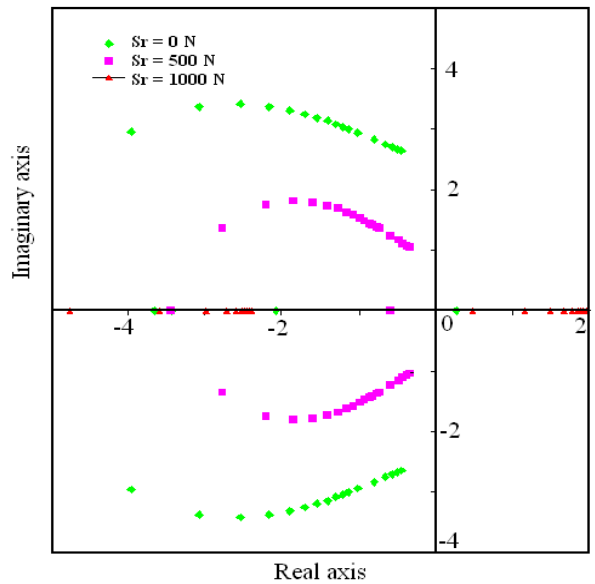

Fig. 7: Root locus at various running speeds under different braking conditions.

The response of the roll angle for a desired lean under various braking conditions are shown for running speed of 40 km/hr (fig.8) and 50km/hr (fig.9). At a running speed of 40 km/hr the two-wheeler shows good stability behaviour during a steady cornering without the application of any braking force (fig.8).

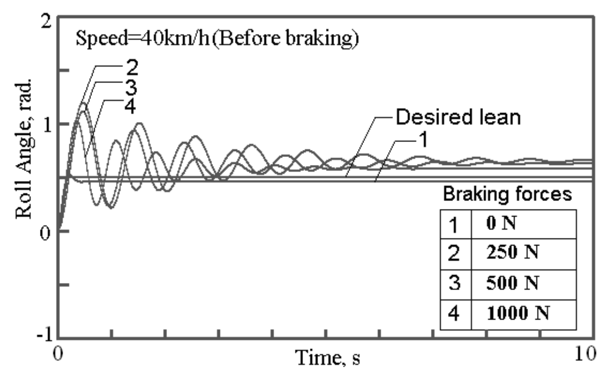

Fig. 8: Roll response of the two-wheeler as it negotiates a turn with and without braking forces.

The stability of the two-wheeler is seriously affected when the braking forces are introduced. It is also observed that peak amplitude of roll decreases to some extent with the application of increased braking forces. Moreover the roll oscillation seems to settle faster with an increase in braking force and the application of appropriate torque on the handle bar. Similar observations are made for a higher running speed of 50 km/hr (fig.9). These observations are important for deciding the safety of the rider because an increase in the peak amplitude may cause the wheel to skid if sufficient support is not obtained from road and tire interaction.

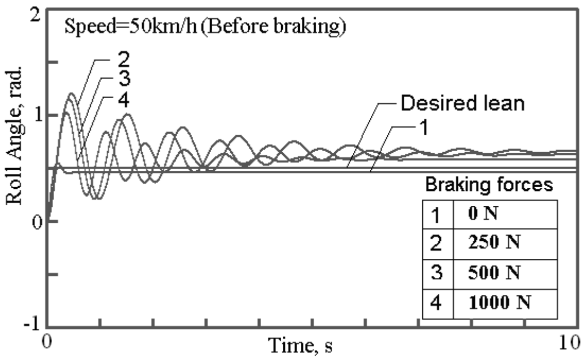

Fig. 9: Roll response of the two-wheeler as it takes a turn with and without braking forces.

Taking the fact that the rider acts as a PID controller, the roll oscillation characteristics can be much improved with a substantial (nearly six times) increase in the derivative gain (fig.10).

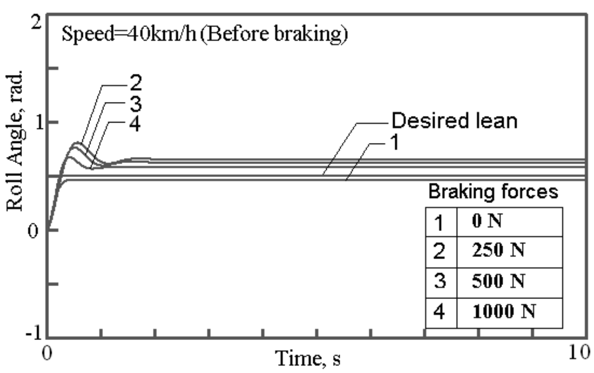

Fig. 10: Roll response with an increased derivative gain for handle bar torque.

It is observed that some of the parameters, e.g., the mass and the trail at the front wheel of the two-wheeler contribute significantly towards its stability. Tire characteristics together with road conditions are also important in this regard. Effects of change in mass and change in trail towards roll stability behaviour is shown in fig.11 for a running speed of 40 km/h under the condition of a braking force of 500 N. As a comparison the above cases are also shown without the application of any braking force.

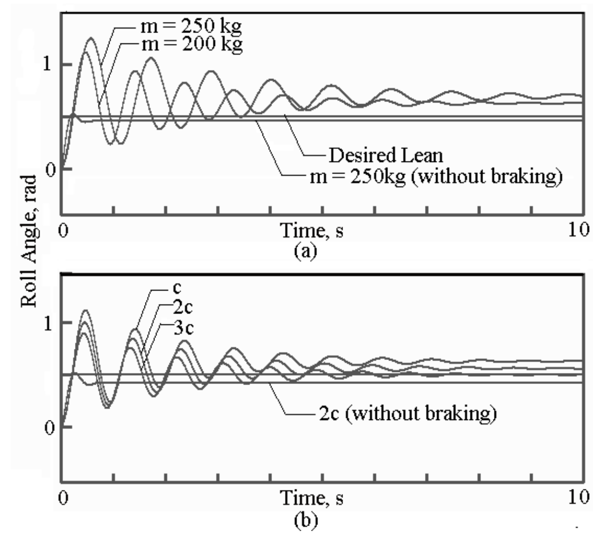

Fig. 11: Effect of parameter changes on the roll behaviour at a speed of 40 km/h (a) change of mass, (b) change of trail.

5 Conclusions

The present paper considers a simplified model of a two-wheeler assuming only its roll freedom. The analysis reveals important facts about running the two-wheeler in a curve especially when a braking force is applied on it. To balance the centrifugal force and the gravity force a two-wheeler in a turn must lean towards the centre of turn. This is essential for ensuring the stability of the two-wheeler. The time responses show that roll behaviour worsen with the braking force applied during a turn. The condition becomes even more worse if the mass of the two-wheeler increases (with a pillion rider) or if the front wheel trail is reduced. But as the peak amplitude is less with harder braking force, stability of the two-wheeler in a curve under such condition of hard braking may be better if proper road condition is met.

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Nomenclature

N_f	Normal force at front wheel, N
N_r	Normal force at rear wheel, N
F_{cf}	Effective centrifugal force on front wheel, N
F_s	Lateral force resisting side slip, N
F_i	Inertia force due to braking, N
S_f	Front braking force, N
S_r	Rear braking force, N
C_λ	Cornering stiffness, N/rad.
C_Φ	Camber stiffness, N/rad.
m	Mass, kg
g	Gravitational acceleration, m/s^2
λ	Head angle, rad
δ	Steer angle with zero trail, rad
δ_f	Effective steer angle, rad
Φ	Roll angle, rad
Φ_f	Front fork roll angle, rad
h	Position of C.G in plane containing rear wheel, m
a	Position of C.G along X-axis, m
b	Length of wheel base, m
c	Trail, m
ψ	Orientation of rear wheel plane, rad
T	Torque applied to handle bar, Nm
θ	Sideslip angle, rad
v	Forward velocity, m/s
f	Deceleration during braking, m/s^2
K_p, K_d, K_i	Controller gains