Modeling of Damage Mechanism in Mouse Skin

S. Sarangi[1*]
[1] Department of Mechanical Engineering, I I T Patna, Patna, India

[*] Corresponding author (email: somsara@iitp.ac.in)

Abstract

A thermodynamically consistent formulation of deformation processes of continua with dislocation motion and defect evolution in the material space on meso- and micro- level is shown. The balance laws of material forces together with the classical balance laws of physical forces and couples, first and second laws of thermodynamics for physical and material space and general constitutive equations are the basis to develop a thermodynamically consistent framework of damage. In this case the local balance laws of material forces together with the constitutive equations represent evolution laws of the material forces. The general form of derived constitutive equation is subsequently simplified through additional assumptions which are relevant to validate uniaxial extension of Mouse skin performed by others. The damage induced stress-softening is characterized by Zúñiga-Beatty front factor damage function. Finally a new phenomenological function is proposed for permanent set in isotropic, isothermal hyperelastic material like skin tissue.

Keywords: Continuum damage mechanics, Material forces, Physical forces, Dissipative processes, Mullins Effect and Permanent Set

1 Introduction

Skin is the largest single organ of the Mammalians body and it comprises of 15-20% of the body weight. The roles of the skin are protection, temperature regulation and transmission of stimuli. Skin interfaces with the environment and plays major role in protecting the body against pathogens and control the excessive water loss. Skin surgery to treat burn injury, to treat cancerous growths are quite common and frequent. In these treatment process skin may undergo large deformation under repetative loading process. This work is focused to study the damage mechanism in the loading cycles and its modeling.

The skin can be divided in two major layers; the outer layer is the epidermis and the lower one is the dermis. Dermis is approximately 20 times thicker than the epidermis. The layers of the skin are composed of gel-like ground substance embaded within collagen fibers, elastin fibers and neuron-fibrils, the TEM image of that was taken by C. Storm *et. al.* [1]. They also experimentally obtained the dynamic shear storage modulus of collagen, actin and neurofilaments. It is observed in [1] that neurofilaments are less elastic than the collagen fibers, however they becomes considerably stiff for larger deformation. Thus in general the elastic response of the skin tissue is primarily governed by the collagen fibers which has low extensibility. Skin poses an oriented fibrous structure and we may consider that structure as an anisotropic structure and this complex structure allows large elastic deformations.

The large deformation in skin induces alterations in the complex orientation of the collagen fibers, elastin fibers and in the neuron network. These non-affine deformation process along with the micro- and meso- level changes are responsible for reduced stress-response in cyclic loading commonly known as Mullins Effect [2] and permanent set on the removal of load. This work formulates a thermodynamically consistent deformation processes of continua with dislocation motion and defect evolution in the material space on meso- and micro-level. Finally, the constitutive equation obtained from the formulation is matched with the experimental data as well as the experimental procedure followed by M. J. Muñoz *et. al.* [3]. It is worth mentioning in this context that modeling of softening phenomenon of mouse skin tissue is recently studied by A. E. Ehret and M. Itskov [4]. In their [4] model anisotropic softening is considered by means of monitoring the evolution of internal variables governing the anisotropic properties of the material. In order to model the simple uniaxial extension one should have the knowledge of nine material parameters. Sometimes they miss the physical significance to study the damage mechanism. In this work a simple thermodynamically consistent damage model of continua is presented which is in line with the experimental procedure of M. J. Muñoz *et. al.* [3]. In this model the assumptions which are made has physical resemblance to the experiment performed by M. J. Muñoz *et. al.* [3]. The present work is based on the generalized theory of continuum mechanics involving balance laws of micro-forces, in addition to the classical laws of mechanics. A single scalar variable, referred as damage variable is introduced in Section 2. The damage variable represents the macro-, meso- and micro level changes in the material. In Section 3 we will satisfy the first law of thermodynamics and the dissipation inequality. In the same section, we will consider the general form of the constitutive equations which are compatible with the dissipation inequality. The generalized form of constitutive relation presented in Section 3, is the basis to impose certain physically justified assumptions. Section 4 imposes the assumption of the isothermal deformation process which relevant to the experiment performed by M. J. Muñoz *et. al.* [3]. To experimentally distinguish the damage from other inelastic effects one chooses the quasi-static loading scheme. Thus the assumption of quasistatic loading is proposed in Section 4 and a simplified form of energy potential is achieved. Finally a few physically valid assumptions are imposed in the Section 5 and a new phenomenological model is introduced.

2 Balance Laws in Physical and Material Space

In this work we apply a Lagrangian formulation of all equations, in turn all fields are referenced to the undeformed and homogeneous reference configuration. The corresponding Eular formulation can be obtained easily by suitable transformation of all quantities to actual configuration (See Stumpf and Hoppe [5]). We will first distinguish the kinematical variables in the physical space and the same in the material space.

Let, us consider a deformable body $\mathcal{B}$ consisting a set of particle p_k i.e. $\mathcal{B}$= { p_k }.The motion of $\mathcal{B}$ is given by a smooth vector function $\mathbf{x} = \chi(\mathbf{X}, t)$ where $\mathbf{X}$ and $\mathbf{x}$ denote the reference and current co-ordinate respectively. Thus one-one mapping of the continuous medium can be represented in terms of the displacement vector $\mathbf{u}(\mathbf{X}, t)$ as

$$\mathbf{x} = \chi(\mathbf{X}, t) = \mathbf{X} + \mathbf{u}(\mathbf{X}, t). \tag{1}$$

The deformation gradient $\mathbf{F}$ is defined as $\mathbf{F} = \dfrac{\partial \mathbf{x}}{\partial \mathbf{X}}$ and the motion (1) is also assumed to be invertible and, consequently, the inverse function $\mathbf{F}^{-1}$ exists. Hence J ≡ det(**F**) is assumed positive. Unlike [3] in this work we introduce a certain variable, which represent underlying physical mechanisms like microstructure changes in a material during the deformation process. It proves effective to use a scalar variable $\alpha = \alpha(\mathbf{X},\ t)$ referred to as damage variable in addition to the gross motion of the body defined by $\mathbf{x} = \chi(\mathbf{X}, t)$. By choosing damage variable as a scalar function, helps one to satisfy the objectivity requirement (Truesdell and Noll [6]) for the constitutive equations.

We will now take a close look on the microforces incorporating the additional softening parameters. Let us assume that the forces accompanying changes of the softening variables α are represented by a (vector) micro stress $\boldsymbol{\pi}(\mathbf{X}, t)$, which characterizes forces, transmitted across internal surfaces. A scalar microforce takes care the internal forces distributed over the volume of the material and represented as $\xi(\mathbf{X}, t)$. Finally the external scalar microforce $\beta(\mathbf{X}, t)$, which may takes care the possible inertia microforce. These three terms $\boldsymbol{\pi}, \xi$ and β vanishes, as there is no change in the damage variables. So, the damage variable may takes care the damage in the skin tissue and by this variable we may describe all the deformation induced changes in the skin tissues. Next we employ the classical balance laws at any part of the body $\mathcal{B}$ such that $P \in \mathcal{B}$ with an element volume dv and element area da at ∂P. However, one can obtain the balance laws of micro-forces along with the classical physical forces by employing the principle of virtual power (See Germain [7].). The principle of virtual power asserts that at any part of the body $\mathcal{B}$ such that $P \in \mathcal{B}$, the virtual power expended on P by materials or bodies exterior to P (*i.e.* external power) be equal to the virtual power expanded within P (*i.e.* internal power). The virtual power balance yields the following relations:

$$\begin{aligned}
&\mathrm{Div}\mathbf{T} + \mathbf{b} = \rho_0 \dot{\mathbf{v}} \text{ (physical force balance)}\\
&\mathbf{t} = \mathbf{T}\mathbf{n} \text{ (physical traction condition)}\\
&\mathrm{Div}\boldsymbol{\pi} - \xi + \beta = \rho_0 \ddot{\alpha} \text{ (micro-force balance)}\\
&e = \boldsymbol{\pi}.\mathbf{n} \text{ (micro-traction condition).}
\end{aligned} \tag{2}$$

where $\mathbf{b} = \mathbf{b}(\mathbf{X}, t)$ is the body force per unit volume in the reference configuration, $\mathbf{T}$ denotes the first Piola-Kirchhoff stress tensor, $\mathbf{n}$ denotes the out ward unit normal vector to the ∂P of P, $\mathbf{v}$ denotes the material velocity and e denotes the surface external microforce of damage.

We will discard the acceleration $\dot{\mathbf{v}}$ in the physical space and the rate of change of $\dot{\alpha}$. Incorporating only the rate effect and form relation (2) one obtains

$$\begin{aligned}
&\int_P (\mathbf{T}.\nabla\mathbf{v} + \xi\dot{\alpha} + \boldsymbol{\pi}.\nabla\dot{\alpha})dv = \\
&\int_P (\mathbf{b}.\mathbf{v} + \beta\dot{\alpha})dv + \int_{\partial P} (\mathbf{T}\mathbf{n}.\mathbf{v} + (\boldsymbol{\pi}.\mathbf{n})\dot{\alpha})da.
\end{aligned} \tag{3}$$

The relation (3) essentially represents the weak form of the microforce balance in any part of the body ∂P of P. The expression on the left hand side of relation (3) represents the stress power density and represented as

$$\sigma = \mathbf{T}.\nabla\mathbf{v} + \xi\dot{\alpha} + \boldsymbol{\pi}.\nabla\dot{\alpha} \equiv \mathbf{T}.\dot{\mathbf{F}} + \xi\dot{\alpha} + \boldsymbol{\pi}.\nabla\dot{\alpha}, \tag{4}$$

where $\nabla\mathbf{v} = \dot{\mathbf{F}}$. The relation (4) depict that if there is no change in the damage variable the stress power density identically satisfy the classical relation $\sigma = \mathbf{T}.\dot{\mathbf{F}}$. Note that the physical force balance equation (2) obtained through the principle of virtual power can be obtained by the classical force balance. Subsequently, the moment balance condition asserts $\mathbf{T}\mathbf{F}^T = \mathbf{F}\mathbf{T}^T$.

3 Dissipation Inequality and Constitutive Relation

A reliable formulation of the constitutive relation of deformable bodies undergoing damage requires a thermodynamic theory for deformation which will account the micro and macro change as well as heat flux in the material space. Any constitutive relation has to satisfy the first and second law of thermodynamics. To validate any constitutive relation one has to satisfy the dissipation inequality. We start the section from the first law of thermodynamics, which gives the energy balance in thermo-mechanical domain and requires the global balance laws of physical forces, couples and material forces. Taking into account the balance of energy in physical and material space as (See Germain [7])

$$\begin{aligned} \frac{d}{dt}\int_P (\varepsilon+\kappa)dv &= \int_P (\mathbf{b}.\mathbf{v}+\beta\dot{\alpha})dv \\ &+\int_{\partial P}(\mathbf{Tn}.\mathbf{v}+(\boldsymbol{\pi}.\mathbf{n})\dot{\alpha})da+\int_P r dv-\int_{\partial P}\mathbf{q}.\mathbf{n}da, \end{aligned} \tag{5}$$

where $\varepsilon(\mathbf{X},t)$ denotes the internal energy, $\kappa(\mathbf{X},t)$ the kinetic energy, $r(X.t)$ the heat source and $\mathbf{q}(X,t)$ the heat flux vector.

The second law of thermodynamics express the principle of entropy growth and its general form is

$$\int_P d\ dv \equiv \frac{d}{dt}\int_P \eta dv-\int_P \frac{r}{\theta}dv+\int_{\partial P}(\frac{\mathbf{q}}{\theta}).\mathbf{n}da \geq 0, \tag{6}$$

where $d(\mathbf{X},t)$ is the energy dissipation per reference volume, $\eta(\mathbf{X},t)$ the specific entropy and $\theta(\mathbf{X},t)>0$ the absolute temperature.

The localised balance of the local equation in material frame which takes care of the change in skin tissue or network alternation thus given below

$$\dot{\varepsilon}=\sigma+r-\mathrm{Div}\mathbf{q}, \tag{7}$$

where σ denotes the stress power density. The second-law of thermodynamics for the local energy balance leads to

$$d=\dot{\eta}-\frac{r}{\theta}+\mathrm{Div}(\frac{\mathbf{q}}{\theta})\geq 0. \tag{8}$$

With the use of local energy balance equation in (7) and definition of free energy $\psi(\mathbf{X},t)$

$$\psi \equiv \varepsilon-\theta\eta, \tag{9}$$

the term "$r-\mathrm{Div}\,\mathbf{q}$" may be eliminated from (8) to give the local dissipation inequality in the form

$$d \equiv \sigma-\eta\dot{\theta}-\frac{q}{\theta}.\nabla\theta-\dot{\psi}\geq 0. \tag{10}$$

The inequality (10) represents the second law of thermodynamics under the assumption that the balance laws of physical and material forces holds. The constitutive equation in (10) is the most general one and it is valid for any physical system undergoing damage. However, the objectivity and the polyconvexity are not checked so far. We have introduced the damage variable as a scalar function in section 2 and it readily complies the objectivity requirement. The polyconvexity of strain energy will be complied later in section 5. In order to study existing experimental results done by Muñoz *et. al.* [3], we will correlate the experimental procedure with the general constitutive relation (10) to look for further simplification.

4. Additional Assumptions relevant to the Experiment performed by Muñoz *et. al.* [3]

In the experiment conducted by Muñoz *et. al.* [3], they anesthetized (n=6) mice by sodium pentobarbital and in the process the body temperature may reduce to 25° C from the normal temperature of 30-32° C. In the dissection process enough precaution was taken to prevent the dehydration of the skin samples. During test ultrasonic dehumidifier was used also and a constant temperature of 25° C was maintained. Thus we lose no generality by assuming the deformation process as isothermal one. Even if one consider the rate of loading, which was very small (15 mm/min) the loading rate does not change the absolute temperature of the material. The assumption of isothermal process reduces the expression (10) in the following way

$$d=\sigma-\dot{\psi} \tag{11}$$

where σ represent the stress power density given in relation (4). The above relation can be recast in a convenient way as

$$d \equiv \mathbf{T}.\dot{\mathbf{F}}+\boldsymbol{\pi}.\nabla\dot{\alpha}+\xi\dot{\alpha}-\dot{\psi}\geq 0 \tag{12}$$

which forms a basis for the subsequent development of a constitutive theory.

It follows from the dissipation inequality relation that the constitutive equation consist of four dynamical variables namely $\psi,\mathbf{T},\boldsymbol{\pi}$ and ξ. The constitutive equation for these variables are the variable of $\mathbf{F}$ and α along with their special and time derivatives of any order. We assume that we will deal with the kind of material where the four dynamic variables $\psi,\mathbf{T},\boldsymbol{\pi}$ and ξ are defined as follows

$$\begin{aligned} \psi &= \tilde{\psi}(\mathbf{F},\alpha,\nabla\alpha,\dot{\mathbf{F}},\dot{\alpha},\nabla\dot{\alpha}), \\ \mathbf{T} &= \tilde{\mathbf{T}}(\mathbf{F},\alpha,\nabla\alpha,\dot{\mathbf{F}},\dot{\alpha},\nabla\dot{\alpha}), \\ \boldsymbol{\pi} &= \tilde{\boldsymbol{\pi}}(\mathbf{F},\alpha,\nabla\alpha,\dot{\mathbf{F}},\dot{\alpha},\nabla\dot{\alpha}), \\ \xi &= \tilde{\xi}(\mathbf{F},\alpha,\nabla\alpha,\dot{\mathbf{F}},\dot{\alpha},\nabla\dot{\alpha}). \end{aligned} \tag{13}$$

Introducing (13) into the second law of thermodynamics (12) leads to the thermodyna-mically admissible form of constitutive equation, where the free energy has to satisfy the following restriction

$$\partial_{\dot{F}}\tilde{\psi}=0,\partial_{\dot{\alpha}}\tilde{\psi}=0,\partial_{\nabla\dot{\alpha}}\tilde{\psi}=0 \tag{14}$$

which means that the free energy may be of the form

$$\psi=\psi(\mathbf{F},\alpha,\nabla\alpha). \tag{15}$$

To simplify the notation scheme, we collect the physical and material kinematical variables and their rates in the ordered sets

$$\begin{aligned} \mathbf{e} &= \{\mathbf{F},\alpha,\nabla\alpha\}, \\ \dot{\mathbf{e}} &= \{\dot{\mathbf{F}},\dot{\alpha},\nabla\dot{\alpha}\}. \end{aligned} \tag{16}$$

With (13) and from the reduced form of free energy in

(15) the dissipation inequality reduced to

$$d = (\tilde{\mathbf{T}}(\mathbf{e},\dot{\mathbf{e}}) - \partial_{\mathbf{F}}\tilde{\psi}(\mathbf{e})).\dot{\mathbf{F}} + (\tilde{\boldsymbol{\pi}}(\mathbf{e},\dot{\mathbf{e}}) \\ -\partial_{\nabla\alpha}\tilde{\psi}(\mathbf{e})).\nabla\dot{\alpha} + (\tilde{\xi}(\mathbf{e},\dot{\mathbf{e}}) - \partial_{\alpha}\tilde{\psi}(\mathbf{e}))\dot{\alpha} \geq 0 \quad (17)$$

A close look on the inequality described in (17) depicts the result that the physical stresses consist of two parts, a non-dissipative part, which can be derived from a free energy potential function, ψ and a dissipative part. The material response can be rewritten in the following way with indicated subscripts `s' for the dissipative part as

$$\begin{aligned} \mathbf{T} &= \partial_{\mathbf{F}}\tilde{\psi}(\mathbf{e}) + \tilde{\mathbf{T}}_S(\mathbf{e},\dot{\mathbf{e}}) \\ \boldsymbol{\pi} &= \partial_{\nabla\alpha}\tilde{\psi}(\mathbf{e}) + \tilde{\boldsymbol{\pi}}_S(\mathbf{e},\dot{\mathbf{e}}) \\ \xi &= \partial_{\alpha}\tilde{\psi}(\mathbf{e}) + \tilde{\xi}_S(\mathbf{e},\dot{\mathbf{e}}) \end{aligned} \quad (18)$$

where the response function with subscripts `s' has to satisfy the dissipation inequality

$$d = \tilde{\mathbf{T}}_S(\mathbf{e},\dot{\mathbf{e}}).\dot{\mathbf{F}} + \tilde{\boldsymbol{\pi}}_s(\mathbf{e},\dot{\mathbf{e}}).\nabla\dot{\alpha} + \tilde{\xi}_s(\mathbf{e},\dot{\mathbf{e}})\dot{\alpha} \geq 0. \quad (19)$$

In (19) the first term denotes the dissipation due to viscous effects on the macrolevel, the second is for the dissipation due to the dislocation in the material space, finally the third term is for the local driving force on the net work defects.

The material response defined in (19) can be rewritten in set form as

$$\boldsymbol{\chi} = \partial_{\mathbf{e}}\tilde{\psi}(\mathbf{e}) + \boldsymbol{\chi}_s(\mathbf{e},\dot{\mathbf{e}}) \quad (20)$$

where the set of dissipative driving forces

$$\boldsymbol{\chi}_s = \tilde{\boldsymbol{\chi}}_s(\mathbf{e},\dot{\mathbf{e}}) = (\tilde{\mathbf{T}}_S(\mathbf{e},\dot{\mathbf{e}}), \tilde{\boldsymbol{\pi}}_s(\mathbf{e},\dot{\mathbf{e}}), \tilde{\xi}_s(\mathbf{e},\dot{\mathbf{e}})) \quad (21)$$

recast the dissipation inequality defined in (19) as

$$d = \tilde{\boldsymbol{\chi}}_s(\mathbf{e},\dot{\mathbf{e}}).\dot{\mathbf{e}} \geq 0. \quad (22)$$

The relation (22) describes the rate of entropy growth and for such a case if it is integrable, then there exist a scalar valued function $\varphi = \tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})$ such that

$$d(\mathbf{e},\dot{\mathbf{e}}) = (\partial_{\dot{\mathbf{e}}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})).\dot{\mathbf{e}} \quad (23)$$

where $\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})$ is a dissipative pseudo potential, which enables dissipative driving stresses (21) by one functional

$$\tilde{\boldsymbol{\chi}}_s(\mathbf{e},\dot{\mathbf{e}}) = \partial_{\dot{\mathbf{e}}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}}). \quad (24)$$

Introducing (24) into (20) leads to thermodynamically consistent constitutive equations

$$\boldsymbol{\chi} = \partial_{\mathbf{e}}\tilde{\psi}(\mathbf{e}) + \partial_{\dot{\mathbf{e}}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}}). \quad (25)$$

The governing equations determining the dissipative deformation process are obtained by introducing the constitutive equation (25) into the physical force balance $(2)_1$ and micro-force balance $(2)_3$ by neglecting the accleration terms, yields the set of equations

$$\begin{aligned} &\mathrm{Div}(\partial_{\mathbf{F}}\tilde{\psi}(\mathbf{e}) + \partial_{\dot{\mathbf{F}}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})) + \mathbf{b} = \mathbf{0}, \\ &\mathrm{Div}(\partial_{\nabla\alpha}\tilde{\psi}(\mathbf{e}) + \partial_{\nabla\dot{\alpha}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})) \\ &-(\partial_{\alpha}\tilde{\psi}(\mathbf{e}) + \partial_{\dot{\alpha}}\tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})) + \beta = \mathbf{0}. \end{aligned} \quad (26)$$

The kinematical variables along with their rates defined in (16) are now to be satisfied by objectivity requirements. The softening variable $\alpha(\mathbf{X},t)$ is a scalar function and satisfies the same. However, the case is not so for $\mathbf{F}$ and $\dot{\mathbf{F}}$. Thus, we have to replace them by their objective counter parts, which can be obtained by pullback to the undeformed and homogeneous reference configuration (See Stumpf and Hoppe [8].). Depending upon the choice of damage variable the $\nabla\alpha$ and $\nabla\dot{\alpha}$ may satisfy the objectivity requirements. But if it does not satisfy the same, then one employs the same technique as discussed earlier. An example of the same is discussed in Section 7.

We have as many equations $(26)_1$ and $(26)_2$ as we have independent unknown fields, $\{\mathbf{F},\alpha\}$ and since the set of equations is complete, we are able to determine the unknown fields supplemented by corresponding boundary and initial conditions. In our consideration so far we have assumed that the free energy $\psi = \tilde{\psi}(\mathbf{e})$ and $\varphi = \tilde{\varphi}(\mathbf{e},\dot{\mathbf{e}})$ are differentiable function. However, they can be sub-differentiable function too.

Muñoz *et. al.* [3] performed the uniaxial extension of Mouse skin specimens under displacement control way. They performed monotonic loading of the specimens at a displacement rate of 15 mm/min. For the gauge length of 25 mm the strain rate reduces to 0.6 /min or 0.01/s. The rate of displacement or the strain rate was small in the experiment. In the experiment major emphasis was given to study the damage and inelastic aspects. In this context we may neglect the rate of loading effect and assume that the loading was quasi-static for the further development of our theoretical model. In this way one avoids the viscous effect along with the other rate dependent effects. The viscous effects on the macro level and physical rate contribution on meso and micro level is neglected through quasistatic loading assumption. Thus for quasi-sataic loading we may set $\dot{\mathbf{F}} = 0, \dot{\alpha} = 0$ and $\nabla\dot{\alpha} = 0$. Thus the set of equation in $(26)_1$ and $(26)_2$, following the quasi-static loading assumptions reduces to

$$\begin{aligned} &\mathrm{Div}(\partial_{\mathbf{F}}\tilde{\psi}(\mathbf{e})) + \mathbf{b} = \mathbf{0}, \\ &\mathrm{Div}(\partial_{\nabla\alpha}\tilde{\psi}(\mathbf{e})) - \partial_{\alpha}\tilde{\psi}(\mathbf{e}) + \beta = \mathbf{0}. \end{aligned} \quad (27)$$

Through this assumption the dissipation inequality defined in (19) is satisfied identically to zero. Still there could be other kind of dissipation in the process of loading. To propose a simple form of constitutive relation we assume that the contributions of dissipative components are negligibly small. So, the stress response defined in (18) can be deduced for the physical co-ordinate in terms of first Piola-Kirchoff stress as

$$\mathbf{T} = \partial_{\mathbf{F}}\tilde{\psi}(\mathbf{F},\alpha,\nabla\alpha). \quad (28)$$

The constitutive equations will have to be reduced further to comply with the principle of material frame-indifference. Such a reduction of constitutive equations is standard and it is not our concern here.

The uniaxial tensile test of the skin specimens may be represented by the deformation mapping as $x_i = \lambda_i X_i$, where x_i are the current coordinates, X_i are the reference coordinates and λ_i are the stretches. Employing the incompressibility constrain the deformation gradient reduces to

$$\mathbf{F} = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda^{-1/2} & 0 \\ 0 & 0 & \lambda^{-1/2} \end{bmatrix}, \quad (29)$$

where λ is the principal stretch in the direction of loading. It appears that the uniaxial extension is a homogeneous deformation and we obtain $\nabla\alpha = 0$. In view of this, the relations (28) and $(26)_2$ reduces to

$$\begin{aligned} \mathbf{T} &= \partial_{\mathbf{F}}\psi(\mathbf{F},\alpha), \\ \partial_{\alpha}\tilde{\psi}(\mathbf{F},\alpha) &= 0. \end{aligned} \tag{30}$$

From (29) we conclude that the general form of strain energy density function W of a damaged material undergoing isothermal, quasi-static and homogeneous deformation may be represented as

$$W = \tilde{W}(\mathbf{F},\alpha). \tag{31}$$

In this case the softening variable α is a single scalar variable. However, one may induce more such scalar variable identifying each of them as separate entity in order to understand the different inelastic aspects in the physical space. In this case we choose α_1 and α_2 as the two different damage variables and following the same procedure as discussed in Section 2 and from relation (30) we may arrive to

$$\begin{aligned} W &= \tilde{W}(\mathbf{F},\alpha_1,\alpha_2), \\ \partial_{\alpha_1}\tilde{W}(\mathbf{F},\alpha_1,\alpha_2) &= \partial_{\alpha_2}\tilde{W}(\mathbf{F},\alpha_1,\alpha_2) = 0. \end{aligned} \tag{32}$$

So far we have modeled the damage mechanism of mouse skin for simple uniaxial extension and introduced two damage parameters in order to study two different inelastic aspects of the experiment performed by Muñoz *et. al.* [3]. Next we will validate our damage model developed in (32) with the experimental data.

5 Validation with the experimental data

In the cyclic loading process the skin tissue showed the stress-softening effect with residual strain, i.e., permanent set. These two effects are commonly observed in rubberlike materials (See Mulins [2], Zúñiga-Beatty [9]). Rubberlike materials exhibiting stress-softening phenomenon show the selective memory property of keeping the maximum previous ever deformation in its strain history. For any deformation beyond the maximum previous ever deformation, the material updates the current one. The fact is observed in the uniaxial experimental data for different maximum previous deformation and the stress-softened material responses are distinctly different for different degrees of maximum previous ever deformation (See Zúñiga - Beatty [9].). The phenomenology of ideal stress-softened material postulates that the stress-response at the maximum previous ever deformation is identical as that of virgin material. In the experimental results obtained by Muñoz *et. al.* [3] show the similar results as that of rubberlike materials (See Figure 3 (a) and (b) of Muñoz *et. al.* [3]).

It is well established that stretch procedures a quasi irreversible rearrangement of the network comprising collagen, elastin and neuron due to localized nonaffine deformation resulting from short chains reaching the limit of their extensibility. This nonaffine deformation produces a displacement of the network junctions from their initial state, in turn produces some form of rearrangements. The constitutive model developed in (32) may be remodeled further to validate the experimental data obtained in [3].

Recently the author with M. F. Beatty and R. Bhattacharyya [10] developed a constitutive theory for a general class of incompressible, isotropic stress-softening, limited elastic rubberlike materials. The developed model used Zúñiga-Beatty [9] font factor damage function to study small superimposed oscillation about finite static stretch of rubberlike elastomers and aortic tissue strips. It is found that Zúñiga-Beatty [9] font factor damage function shows good agreement to predict the frequency of aortic tissue strip collected from thoracic region. In this damage formulation we set the damage parameter α_1 as the Zúñiga-Beatty [9] font factor to model the stress-softening behavior. It is worth mentioning that the Zúñiga-Beatty [9] font factor is an isotropic damage function and we have discarded $\nabla\alpha_1$ for homogeneous deformation, which essentially means that α_1 is an isotropic function also for simple uniaxial extension. The first damage variable in (32) may be written as

$$\alpha_1 = F_1(s;S) \equiv e^{-b_1\sqrt{S-s}}, \tag{33}$$

where b_1 is the softening parameter and from (29) we obtain $s = \sqrt{\lambda^4 + 2/\lambda^2}$ as the current strain intensity (See [10 -11] for details.). The corresponding function of selective memory is obtained through the maximum previous strain intensity defined by

$$S = \max_{0\le\tau\le t} s(\tau), \tag{34}$$

where the material is subjected to a deformation history up to the current time t, and τ denotes a running time variable (See Zúñiga-Beatty [9] for details.).

The skin samples collected from the abdominal region along the longitudinal length was of 25 mm in length, average of 5 mm in width and average of 0.58 mm in thickness for male specimens and 0.41 for female specimens. The length scale is five times higher than the width scale and the thickness scale is at least 1/43 times the length scale. We assume that the anisotropic model of the skin tissue will have low to moderate contribution in uniaxial extension. To model the parent material we choose Gent[12] type isotropic limited elastic material model defined in accordance to relation (32) as

$$\begin{aligned} W &= \tilde{W}(\mathbf{F},1,1) = \tilde{W}(\mathbf{B},1,1) \\ &= -\frac{G}{2}(I_m - 3)\log\left(1 - \frac{I_1 - 3}{I_m - 3}\right), \end{aligned} \tag{35}$$

where $\mathbf{B} = \mathbf{F}\mathbf{F}^T$ is left Cauchy-Green deformation tensor and G is the rigidity modulus at the ground state. The first invariant of $\mathbf{B}$ is denoted by I_1 and I_m is the limiting value of I_1 at the limiting deformation. The shear response functions are obtained for limited elastic Gent [12] type material as

$$\begin{aligned} \beta_1 &= 2\frac{\partial W}{\partial I_1} = G\left(\frac{1 - 3/I_m}{1 - I_1/I_m}\right) > 0, \\ \beta_{-1} &= 2\frac{\partial W}{\partial I_2} = 0. \end{aligned} \tag{36}$$

The constitutive curve for simple uniaxail deformation of parent Gent [12] type material will be monotonically increasing and for $\lim_{I_1 \to I_m} \beta_1 = +\infty$ and $\lim_{I_m \to \infty} \beta_1 = G$. The functional form of energy defined in (35) satisfies the material frame indifference or objectivity requirement and from condition (36) the energy form satisfies the polyconvexity requirement as well.

Muñoz *et. al.* [3] experimentally observed that the residual strain, i.e., the permanent set in uniaxial deformation had a good correlation with the maximum strain previously obtained which is denoted by *S* in (34). Phenomenological model of second damage variable is defined as

$$\alpha_2 = F_2(s;S) = e^{s/S-1} \leq 1. \tag{37}$$

From the expression of two damage variables defined in (34) and (37) we define the engineering stress response of damage skin in simple uniaxial extension with the help of relation (32) and (36) as (See [10] for detail derivations.)

$$\sigma_s = G\left(\frac{1-\dfrac{3I_m}{I_m}}{1-\dfrac{I_1}{I_m}}\right)\left((\lambda - \frac{1}{\lambda^2})e^{-b_1\sqrt{S-s}} - b_2(1-e^{\frac{s}{S}-1})\right), \tag{38}$$

where b_2 is a nondimensional material constant associated with residual strain. The stress response for monotonic loading may be obtained by setting current strain intensity as maximum previous strain intensity, i.e., $s = S$ in (38) as

$$\sigma_v = G\left(\frac{1-3/I_m}{1-I_1/I_m}\right)(\lambda - \frac{1}{\lambda^2}). \tag{39}$$

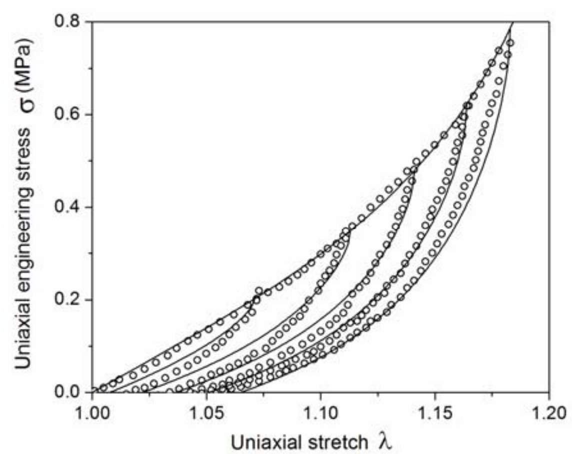

Fig. 1: Comparison of the theoretical monotonic loading (from relation (38)) and cyclic loading (from relation (39)) curves with the Muñoz *et. al.* [3] uniaxial extension data of 18 months old male mouse skin .

For the constitutive model of damaged skin tissue, the uniaxial engineering stress-stretch data for 18 months old male mouse the monotonic loading is first fitted to the equation (39) to obtain the shear modulus $G = 0.942$ MPa and to obtain limiting extensibility constant $I_m = 3.206$ (i.e., limiting stretch $\lambda_m = 1.284$). These values were then used in equation (39) to obtain by a best fit from the cyclic loading data the values $b_1 = 2.487, b_2 = 1.694$ of nondimensional material parameters. The results for the model in (38) and (39) are mapped as solid line in Fig. (1). These curves are close to the data in both monotonic loading and cyclic loading. The damaged paths are deviating slightly in lower stretch but still quite close to the model developed in (39). The experimental data points are shown by small unfilled circles are taken from the fitted data plot of Muñoz *et. al.* [3] in strain controlled manner.

6 Conclusions

A generalized approach considering the physical and material coordinate and the corresponding micro-forces along with the physical forces are presented in order to formulate the damage from the simple theory of continuum damage mechanics. The constitutive model defined in (38) is chosen as simplest form. However, they can be in fractional derivative or may induce other complex form. The assumption of quasi-static loading enables one to distinguish the damage from other inelastic effects such as viscoelastic effects, Payne effects. Finally the proposed softening model defines a pretty simple functional form and the correspondingly the micro-forces are taken care through the physical coordinates by phenomenological postulations. Although the proposed softening function is based on the assumption of isotropic damage, one may obtain the corresponding anisotropic model satisfying the thermo-mechanical constraints discussed in this paper.

The success of our simple damaged material model (38) in characterizing the Mullins effect and residual strain is evident in two respects. First of all it may produce simple closed form analytical solution for the application problems both in Boundary value problems and in dynamical problems. Secondly, comparison of our phenomenological model with the experimental data requires determination of only four material constants: the shear modulus G, the limiting first invariant, i.e. $I_m = I_1|_{\lambda=\lambda_m}$ for Gent [12] type material model and the two nondimensional material constants b_1 and b_2. In contrast, fiber directed anisotropic damage model recently developed by Ehret-Itskov [4] that compare favorably with the experimental data do so by fitting a great many parameters like nine that lack, in most cases, of any physical interpretation at all, and which also demand considerable computation.

Acknowledgment

Author sincerely thanks Prof. Ranjan Bhattacharyya, Department of Mechanical Engineering, IIT Kharagpur for his valuable suggestions to formulate this problem.

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