Static balancing of Spring-loaded Planar Revolute-joint Linkages without Auxiliary Links

Sangamesh R. Deepak and G. K. Ananthasuresh
Multidisciplinary and Multi-scale Device and Design (M2D2) Laboratory
Mechanical Engineering, Indian Institute of Science, Bangalore 560012, India
Email addresses: {sangu, suresh}@mecheng.iisc.ernet.in

Abstract

We present a method to statically balance a general tree-structured, planar revolute-joint linkage loaded with linear springs or constant forces without using auxiliary links. The balancing methods currently documented in the literature use extra links; some do not apply when there are spring loads and some are restricted to only two-link serial chains. In our method, we suitably combine any *non*-zero-free-length load spring with another spring to result in an effective zero-free-length spring load. If a link has a single joint (with the parent link), we give a procedure to attach extra zero-free-length springs to it so that forces and moments are balanced for the link. Another consequence of this attachment is that the constraint force of the joint on the parent link becomes equivalent to a zero-free-length spring load. Hence, conceptually, for the parent link, the joint with its child is removed and replaced with the zero-free-length spring. This feature allows recursive application of this procedure from the end-branches of the tree down to the root, satisfying force and moment balance of all the links in the process. Furthermore, this method can easily be extended to the closed-loop revolute-joint linkages, which is also illustrated in the paper.

Keywords: Spring load, zero-free-length springs, serial chain, composition of springs.

1 Introduction

A statically balanced linkage maintains static equilibrium in all its configurations. Methods reported in the literature for static balancing of serial and parallel linkages involve the addition of springs and links [1-6]. In this paper we show how *without adding any extra link*, one can balance planar, revolute-joint linkages loaded with linear springs.

Static balance of general planar linkages, including revolute-joint linkages, under gravity load was given in [1]. Approximate static balance of planar linkages against gravity using normal springs, i.e., positive free length springs was considered in [2]. Robot manipulators were statically balanced against gravity using cam-pulley in [3]. Static balance of two-link open-loop revolute-joint linkage under *spring load* was given in [4] and [5]. However, there is no method in the literature for a general *spring-loaded* revolute-joint planar linkage. Furthermore, all the above works make use of auxiliary links.

The method presented in this paper uses zero-free-length springs. A *zero-free-length spring* is a linear spring whose length is zero when the spring force is zero. In other words, the deformation is equal to the length. The spring force in a zero-free-length spring is contrasted with a normal linear spring in Fig. 1. Ways to practically realize zero-free-length springs as well as negative-free-length springs were given in [5] and [6]. One such method is described in the Appendix. Furthermore, it was shown in [5] that the gravity load (i.e., a constant force) is a special case of a zero-free-length-spring load.

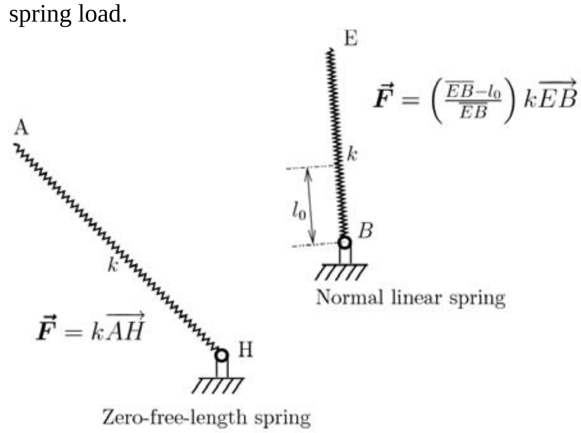


Figure 1: Forces in zero-free-length spring in contrast to a normal linear spring as a function of end points of the spring

The new static balance method we present in this paper requires that all the load springs be of zero free-length. If a load-spring is a positive-free-length spring, then it is suitably combined with a negative-free-length spring in parallel to get an effective zero-free-length spring-load.

We use the concept of composition of springs to develop the method. The concept is that the net effect of several zero-free-length springs with one of their end points connected separately to different points of a link and their other end points anchored *together at a single point* on another link, is equivalent to a single zero-free-length spring connecting a specific point in the first link and the common point on the second link. This concept was used in [5] but in a different context. The proof of this concept is given in Section 2.

Central to our method is a procedure of adding springs to a link. This procedure can be applied to a link on a condition that all the forces on the link, except the parent-joint constraint force, should be zero-free-length spring-loads or equivalents of zero-free-length spring-loads. A two-step composition involving these springs and the ones added during the procedure has two consequences: (i) force and moment balance of the link is satisfied in any configuration, and (ii) the parent joint constraint force is equivalent to a zero-free-length spring load.

The condition on a link to apply the above procedure is readily satisfied by terminal links (which are referred to as child-less links). Once the procedure is applied to all the terminal links, because of the second consequence of the process, few more links down the tree structure satisfy the condition and the procedure can be applied on them. Again, some more links become eligible and this will continue all the way to the root link. In short, the second consequence of the spring-addition process enables the procedure to be applied on all the links recursively. Once the procedure is applied on all the links, the first consequence of the process ensures force and moment balance for all the links at any configuration, leading to static balance of the linkage. The rationale and details of this procedure as applied to open or tree-structured linkages and then its extension to closed-loop linkages is presented in Sections 3 and 4. We conclude with main points in Section 5.

2 Composition of zero-free-length springs

Consider Fig. 2 which shows two rigid bodies (called *links*) numbered 1 and 2 with n zero-free-length springs connected between them. On link 1, the anchor points of all the springs are coincident at P whereas on link 2, the anchor point of spring k_i is at A_i. We now show that these springs are equivalent to a single zero-free-length spring.

$\overrightarrow{OA_i}$ is position vector of point A_i on link 2 with respect to a reference point O on the link. Let A be the point on the link so that

$$\overrightarrow{OA} = \frac{\sum_{i=1}^{n} k_i \overrightarrow{OA_i}}{\sum_{i=1}^{n} k_i} \quad (1)$$

The net spring force at point P on link 1 is given by

$$\vec{F} = \sum_{i=1}^{n}\left(k_i \overrightarrow{PA_i}\right)$$
$$= \sum_{i=1}^{n}\left(k_i\left(\overrightarrow{PO} + \overrightarrow{OA_i}\right)\right)$$
$$= \overrightarrow{PO}\left(\sum_{i=1}^{n} k_i\right) + \frac{\sum_{i=1}^{n}\left(k_i \overrightarrow{OA_i}\right)}{\sum_{i=1}^{n} k_i}\left(\sum_{i=1}^{n} k_i\right)$$
$$= \overrightarrow{PO}\left(\sum_{i=1}^{n} k_i\right) + \overrightarrow{OA}\left(\sum_{i=1}^{n} k_i\right) \text{ (from Eqn. 1)}$$
$$\Rightarrow \vec{F} = \overrightarrow{PA}\left(\sum_{i=1}^{n} k_i\right)$$

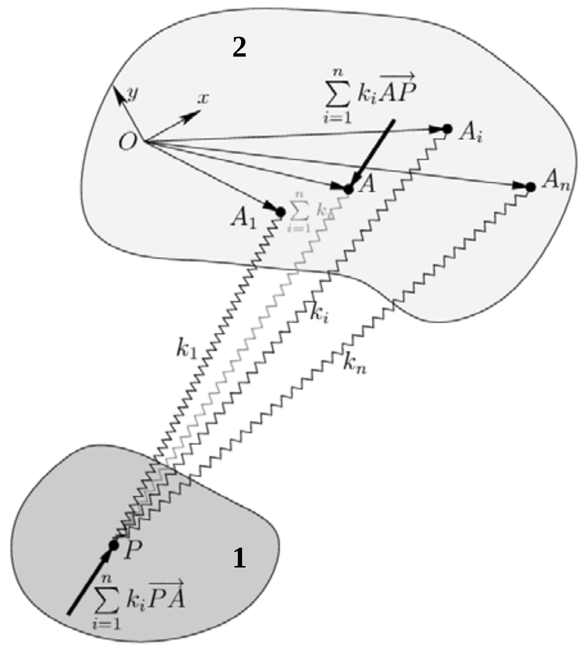


Figure 2: Composition of zero-free-length springs about point P. The resultant spring is shown in gray color.

The net spring force on link 2 is the same but in the opposite direction, i.e., $\overrightarrow{AP}\sum_{i=1}^{n} k_i$. Furthermore, its line of action passes through A and P. Thus, the resultant force acting on the two links can be obtained if we replace the existing springs with a single zero-free-length spring of spring constant $\sum_{i=1}^{n} k_i$ connected between points A on link 2 and point P on link 1. Hence such a spring is equivalent to all of the existing springs and we call this spring the resultant of composition of the existing springs about point P. To emphasize that the resultant spring is not a physically existing spring we draw it in gray color in this and subsequent figures.

3 Static balancing revolute-joint planar linkages

We begin with the static balancing of tree-structured planar revolute-joint linkages loaded with a zero-free-length-spring. We then show how it can be extended to closed-loop linkages.

3.1 Balancing open-chain linkages

Open-chain linkages can have serial-chain or tree-structure configurations. We first illustrate our method on a serial revolute-joint linkage loaded with a single spring on the terminal link. An example of such a linkage is shown in Fig. 3. As noted in the introduction, before we apply the method it should be ensured that all the loading springs are of zero-free-length. All springs that are added should also be of zero-free-length.

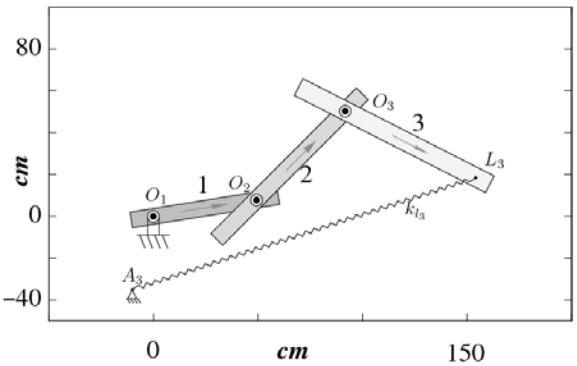


Figure 3: A serial three-link zero-free-length-spring-loaded revolute-joint linkage (arrows indicate x-axis of local co-ordinate system)

In Fig. 3, there are three moving links. Unlike the other links, the terminal link (i.e., link 3) has only one joint and hence only one unknown force (joint-reaction). We first analyze this link for force balance and moment balance *at any configuration* using its free-body diagram shown in Fig. 4.

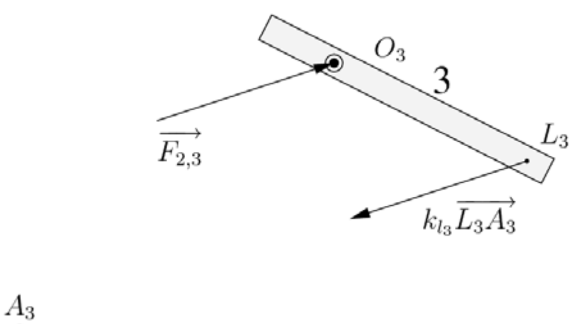


Figure 4: Free body diagram of link 3 of Fig. 3

While the joint-reaction force $\overrightarrow{F_{2,3}}$ could be taken as equal and opposite of the spring force $k_{l_3}\overrightarrow{L_3A_3}$ to satisfy force-balance, an unbalanced couple cannot be avoided at any arbitrary configuration of the link. The only way to avoid this couple is to have the spring-loading point L_3 coincident with the joint at O_3. But we cannot change the loading-point of the given load-spring. So, we add additional springs between link 3 and the ground so that the composition of all the springs on link 3 results in a spring having its anchor at O_3 on link 3. To facilitate spring-composition in further steps, we also want to have the ground-anchor point of this resultant spring to be coincident with the anchor point of the root link (i.e., link 1) at O_1. A resultant spring anchored at O_3 on link 3 and at O_1 on the ground is obtained through a two-step spring composition shown in table 1.

The consequence 2 in Table 1 is significant. Before adding the springs, link 2 had two unknown joint reaction forces: $\overrightarrow{F_{3,2}}$ (child-joint) and $\overrightarrow{F_{1,2}}$ (parent-joint). After adding the springs, $\overrightarrow{F_{3,2}}$ is known and is also the same as a spring load. Therefore, link 2 can now be treated as a terminal link for applying the procedure that we used for link 3. For force and moment balance of link 2, we need to compose the spring in Fig. 5 with an extra spring so that loading point of the resultant spring is at the parent joint O_2. This is again accomplished by adding a spring as shown in Table 2.

The consequence of adding the springs on link 2 is that the child-joint constraint force on link 1 is equivalent to a spring load as shown in Fig. 6. This spring load is countered by the reaction force at the root pivot at O_1 without any couple. Thus link 1 (the root link) also satisfies force and moment balance in addition to other two links (see consequence 1 in Tables 1 and 2). Thus, the entire linkage is in static balance.

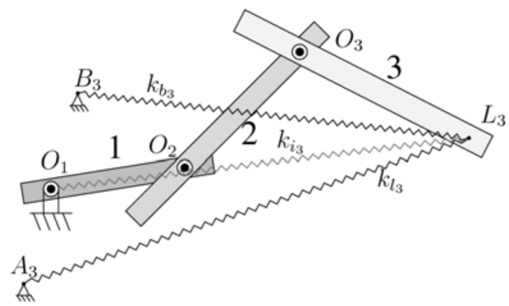
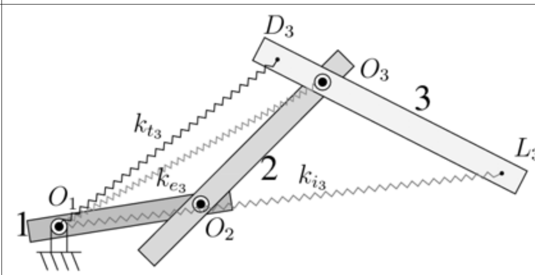
All the real springs that were added are shown in Fig. 7. As a verification of the method, we have varied ϕ_1, ϕ_2, and ϕ_3, the angles of local x-axis of the links with the global x-axis (see Fig. 3), in the parametric form $\frac{\pi}{2}(0.1+\sin t)$, $\frac{\pi}{2}(0.5\cos t)$, and $\frac{\pi}{2}(-0.3+\sin 2t)$, where t varies from 0 to 2π. The potential energy plot of all the springs as well as their sum is shown in Fig. 8. The link lengths are to the scale in Fig. 3. The spring constants were chosen as follows: $k_{l_3} = 1N/cm$; $k_{l_3} = k_{b_3}$; $k_{t_3} = 4k_{i_3}$; $k_{t_2} = 4k_{e_3}$.

The procedure just described is applicable even when the linkage is tree-structured and loaded by multiple springs. To understand this extension, consider a simple tree-structured linkage shown in Fig. 9. Links 3 and 4 are terminal links in it and hence the procedure of statically balancing them is applicable to them. The link 2 will then have equivalent spring forces on it as shown in Fig. 10. Now, it is possible to apply the two-step spring composition procedure on link 2 as illustrated in Table 3. The consequence of this spring-addition is the same as in Table 2 and the rest of the argument for static balance of this linkage is the same as that for the serial linkage of Fig. 3.

The procedure remains the same for any tree-structured linkage. One has to start with terminal links and proceed towards the root and balance all the links in between. When the root link is reached, the entire linkage will be statically balance.

For completeness and clarity, we now summarize the steps of the balancing method to facilitate its application to any tree-structured revolute-joint linkage.

Table 1: Spring adding process on link 3

Aim : Obtain resultant spring from O_3 on link 3 to O_1 on the ground	
First spring-composition Aim: Ground-anchor point of the resultant spring at the root pivot O_1. Links involved: ground and link 3 Springs involved: k_{l_3} and k_{b_3} (extra); Resultant spring : k_{i_3} Common point : L_3 on link 3 Equation of composition : $k_{b_3}\overrightarrow{O_1B_3} + k_{l_3}\overrightarrow{O_1A_3} = \vec{0}$ $k_{i_3} = k_{l_3} + k_{b_3}$	
Second spring-composition Aim: Loading point of the resultant at the parent joint O_3. Links involved: ground and link 3 Springs involved: k_{i_3} and k_{t_3} (extra); Resultant spring : k_{e_3} Common point : O_1 on the ground Equation of composition : $k_{i_3}\overrightarrow{O_3L_3} + k_{t_3}\overrightarrow{O_3D_3} = \vec{0}$ $k_{e_3} = k_{i_3} + k_{t_3}$	
Take joint reaction force on link 3 , $\overrightarrow{F_{2,3}}$, to be equal and opposite of spring force of the resultant , $k_{e_3}\overrightarrow{O_3O_1}$. **Consequence 1**: Force and moment balance of the link satisfied in any configuration.	

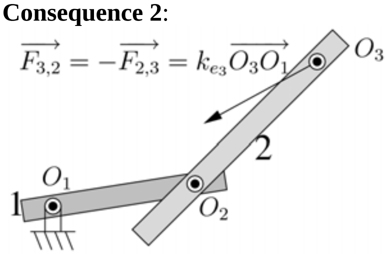
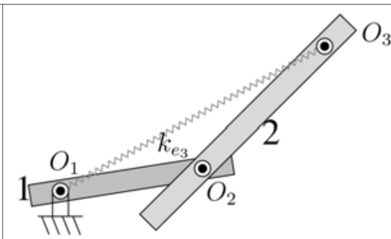
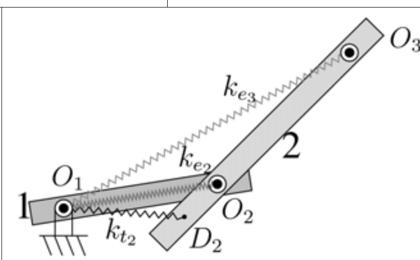
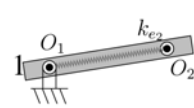
Consequence 2: 	For static analysis of links 2 and 1, the two are equivalent	 **Figure 5: A spring load equivalent to the child-joint constraint force on link 2**

Table 2: Spring adding process on link 2

Aim : Obtain resultant spring from O_2 on link 2 to root pivot O_1 on the ground	
First spring-composition: Ground-anchor point of the resultant spring at the root pivot O_1	Satisfied as it is in Fig. 5
Second spring-composition: Loading point of the resultant at the parent joint O_2. Links involved: ground and link 2 Springs involved: k_{e_3} and k_{t_2} (extra); Resultant spring : k_{e_2} Common point : O_1 on the ground Equation of composition : $k_{e_3}\overrightarrow{O_2O_3} + k_{t_2}\overrightarrow{O_2D_2} = \vec{0}$ and $k_{e_2} = k_{e_3} + k_{t_2}$	
Take joint reaction force on link 2, $\overrightarrow{F_{1,2}}$, to be equal and opposite of spring force of the resultant , $k_{e_2}\overrightarrow{O_2O_1}$.	

Consequence 1: Force and moment balance of the link satisfied in any configuration	**Consequence 2**: The joint reaction force $\overrightarrow{F_{2,1}}$ on link 1 is equivalent to a spring force as shown in Fig. 6	 **Figure 6: Equivalent spring on link 1**

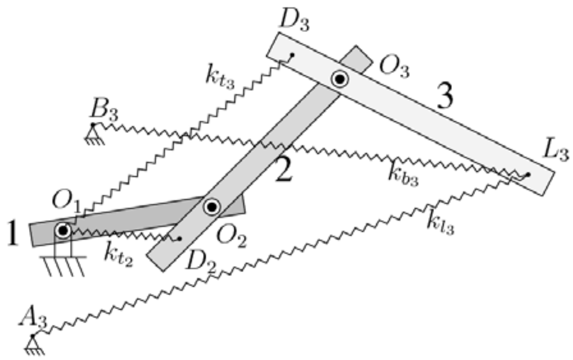


Figure 7: Real springs - loading as well as extra, in the balanced linkage

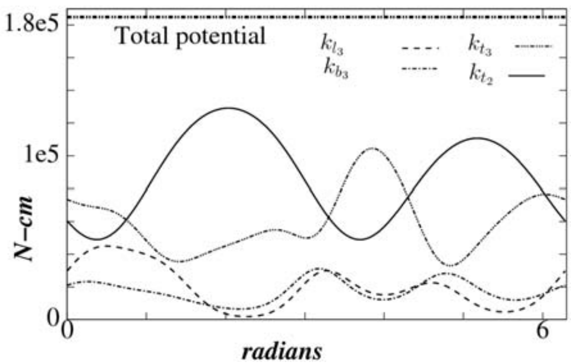


Figure 8: Potential Energy plot of springs and their sum

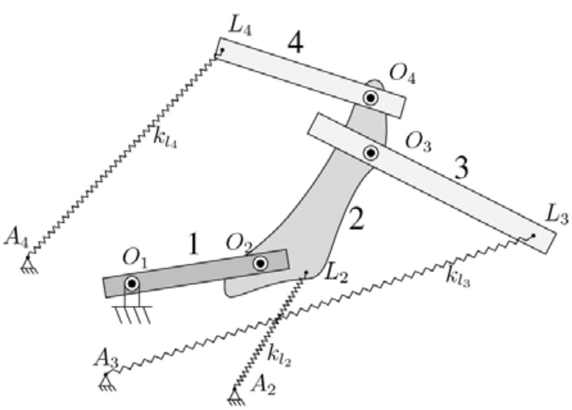


Figure 9: A tree-structured linkage loaded with multiple springs

3.2 Summary of the static balancing method for open chains

In a linkage, if a link satisfies the condition that "all the forces on the link, other than the parent-joint constraint force, are zero-free-length-spring loads or equivalents of zero-free-length-spring loads", then the following two-step spring composition can be carried out.

Step 1: If the ground anchor point of any of the springs (real as well as equivalents) does not coincide with the root pivot, then an additional spring is added so that its composition with the spring about the loading point on the link results in a spring between the loading point and the root pivot. For this, we use the technique described in Section 2.

Step 2: Step 1 results in a set springs all which have their ground anchor point at the root pivot. This set of springs is then combined with an additional spring about the root pivot so that the loading point of the resultant spring on the link is at the parent joint. This spring is equivalent to all the forces (including spring forces added in step 1 and 2) on the link other than the parent-joint constraint force.

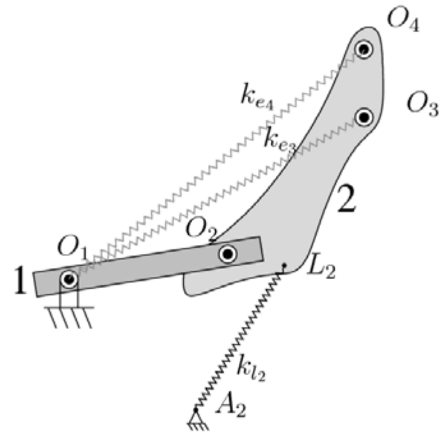


Figure 10: Spring loads equivalent to joint reaction on link 2 after spring addition process on links 3 and 4

The consequences of this two-step spring composition are:

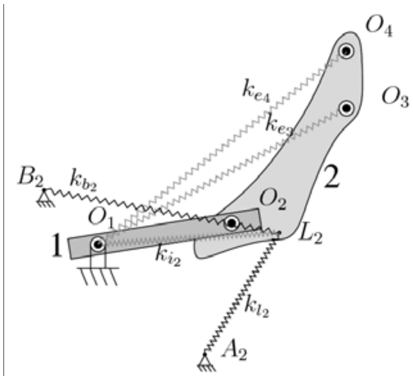
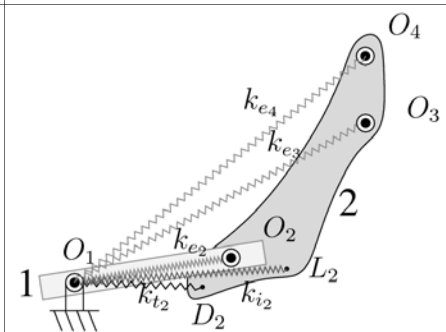
Consequence 1: By taking the parent-joint constraint force to be equal and opposite of the load of the resultant spring (from the step 2 above), equations of static balance associated with the link are satisfied.

Consequence 2: The joint reaction on the parent link is equivalent to a spring load from the parent joint to the root pivot.

Terminal links readily satisfy the condition stipulated on a link to apply the two-step static balancing procedure. Once the process is applied on all terminal links, because of the second consequence, few more links down the tree satisfy the condition. This recursively continues all the way till the root link also satisfies the stipulated condition. Once the root satisfies the condition, we apply step 1 of the two-step spring composition on it. This results in a set of springs (that are equivalent to all forces on it excluding root joint reaction) anchored from the root link to root pivot as shown in Fig. 11.

The root link with springs as shown in Fig. 11 is statically balanced *as it is.* The reason is, the spring loads form a system of concurrent forces with the point of concurrence at the root pivot. The line of action of resultant of those forces always passes through root pivot and is balanced by the ground reaction force without creating any net couple. Thus equations of static balance associated with the root link are satisfied. Thus, step 2 of two-step spring composition is not required for the root link.

Table 3: Spring adding process on link 2

<table>
<tr><td colspan="2">Aim : Obtain resultant spring from O_2 on link 2 to O_1 on the ground</td></tr>
<tr><td>First spring-composition
Aim: Ground-anchor point of the resultant spring at the root pivot O_1.
Links involved: ground and link 2
Springs involved: k_{l_2} and k_{b_2} (extra); Resultant spring : k_{i_2}
Common point : L_2 on link 2
Equation of composition : $k_{b_2}\overrightarrow{O_1B_2} + k_{l_2}\overrightarrow{O_1A_2} = \vec{0}$
$k_{i_2} = k_{l_2} + k_{b_2}$</td><td></td></tr>
<tr><td>Second spring-composition
Aim: Loading point of the resultant at the parent joint O_2.
Links involved: ground and link 3
Springs involved: k_{i_2}, k_{e_3}, k_{e_4} and k_{t_2} (extra);
Resultant spring : k_{e_2}
Common point : O_1 on the ground
Equation of composition :
$k_{i_2}\overrightarrow{O_2L_2} + k_{t_2}\overrightarrow{O_2D_2} + k_{e_3}\overrightarrow{O_2O_3} + k_{e_4}\overrightarrow{O_2O_4} = \vec{0}$
$k_{e_2} = k_{i_2} + k_{t_2} + k_{e_3} + k_{e_4}$</td><td></td></tr>
<tr><td colspan="2">Take joint reaction force on link 3 , $\overrightarrow{F_{1,2}}$, to be equal and opposite of spring force of the resultant , $k_{e_2}\overrightarrow{O_2O_1}$.</td></tr>
<tr><td colspan="2">Consequence 1: Force and moment balance of the link satisfied in any configuration</td></tr>
<tr><td colspan="2">Consequence 2: The same as that in table 2.</td></tr>
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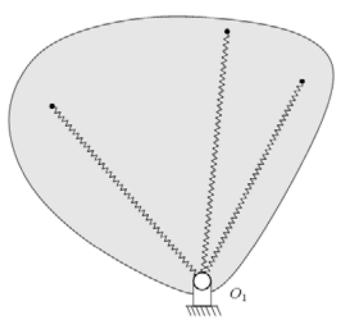

Figure 11: Springs equivalent to all the forces on the root link after step 1 of spring composition involving the link

In the recursive process while coming down the tree to the root, because of consequence 1 of the process, equations of static balance of non-root links were already satisfied. Now with equations of static balance of the root also satisfied, the whole linkage is in static balance.

Thus, any open linkage (serial or more generally, tree-structured) can be statically balanced. This can be extended to closed-loop linkages too, as explained next.

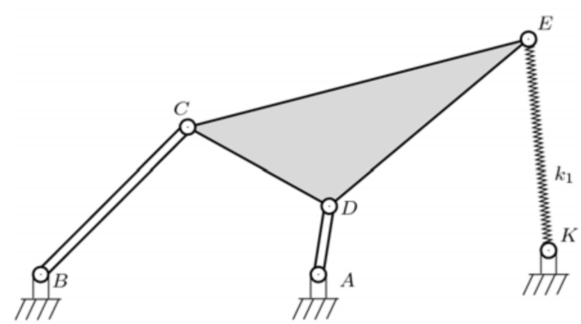

Figure 12: A closed loop linkage: four-bar linkage with a spring load attached to the coupler link

4. Balancing closed loop linkages

A rather simple strategy to statically balance a spring loaded planar closed loop revolute joint linkage, such as the four-bar linkage shown in Fig. 12, is to conceptually break it at certain joints to make it into open-loop linkage as illustrated in Fig. 13. The open-loop linkages are then balanced by the method described in Section 3 and then joints are reconnected back to get the original linkage which would now statically balanced. For the linkages shown in Fig. 13, the unloaded single link doesn't require balancing where as the spring-loaded serial chain of two links is balanced as shown in Fig. 14. Reconnection of the broken joint of the four-bar linkage, after balancing, is shown in Fig. 15.

Thus, the technique presented here is sufficiently general to balance revolute-jointed planar linkage for static equilibrium in all its configurations. It can also be extended to spatial linkages.

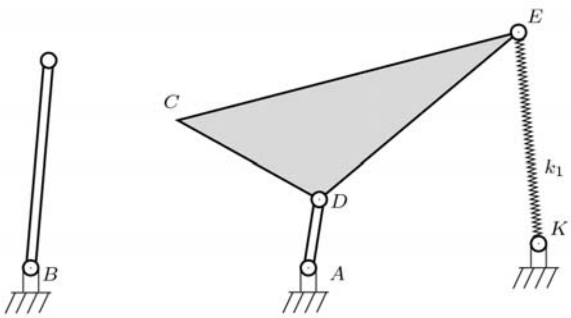

Figure 13: Breaking of joints to make open-loop linkages out of the closed loop linkage

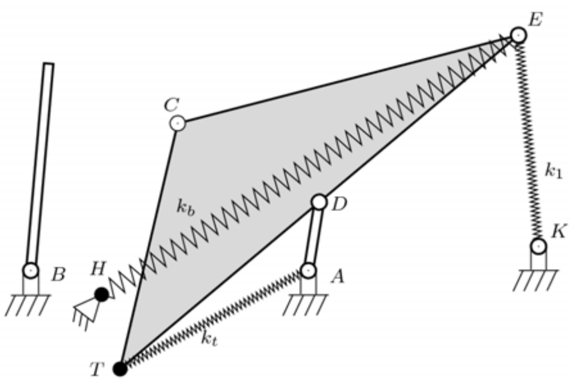

Figure 14: Balancing of two open loop linkages

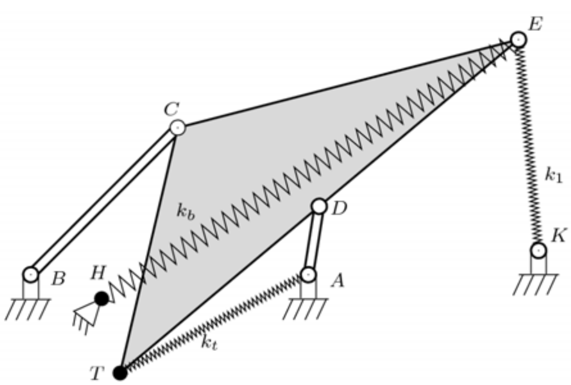

Figure 15: Reconnection of broken joint, post balancing

5 Conclusion

In this paper we presented a new technique for the static balancing of a spring-loaded revolute-jointed planar linkage without adding any auxiliary links. All springs are assumed to have zero free length, which does not preclude its use in practice because such springs can easily be constructed even with non-zero free-length springs. Appendix illustrated one easy way of accomplishing this. The key concept underlying the technique is to add springs to a terminal link to reduce the net force at its free end to a single spring attached to the root link. After the terminal links are balanced, their parent links are balanced in the same way until the root link is reached. We first illustrated our method on a tree-structured open-chain linkage and then showed how it can be extended to close-loop linkage too. We note that this general method can also be extended to spatial linkages.

Acknowledgment

We thank Professor Dibakar Sen, Centre for Product Design and Manufacturing, IISc, whose chance remark that if additional links are added to balance a linkage, we are *actually* balancing another linkage and not the original one, partially motivated this work.

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Appendix

A method to obtain a zero-free-length spring

A zero-free-length spring of spring constant k, with one end anchored to the ground at A and the other (moving) end at B, exerts a force of $k(\overrightarrow{BA})$ at B. Any arrangement which exerts a force of $k(\overrightarrow{BA})$ at B is a zero-free-length spring. One such arrangement is described below.

1) Place a pulley at A, the diameter of which is negligible compared to the length of AB, as shown in Fig. A1.
2) Anchor a normal positive free-length spring at a convenient point P such that $(PA - l_0) > AB$, where l_0 is the free-length of the spring.
3) Attach a string of length $(PA - l_0)$ to the other end of the spring so that when the spring has no deflection, the free end of the string coincides with A.
4) Pass the free end of the string over the pulley and place it at B. This is accompanied by a deflection x of the real spring whose magnitude is the same as the length of AB, as shown in Fig. A1.

The force exerted by the string at B is along $\overrightarrow{BA}$ and of magnitude kx. Since $x = AB$, the force exerted may be written as $k(\overrightarrow{BA})$. Thus a zero-free-length spring of spring constant k anchored at A is realized.

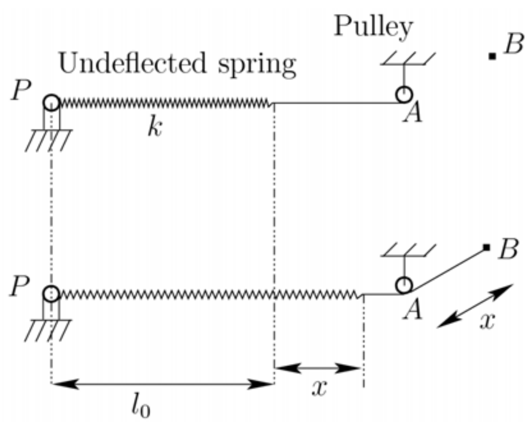

Figure A1: Obtaining a zero-free-length spring from a positive free-length spring, i.e., from a normal spring