Design and Control of Biped Robot

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Abstract

Biped robots have been an interesting research topic for many years, and recent developments in actuators, sensors and computers have enabled the realization of more and more sophisticated anthropomorphic biped robots. Study of the human gait patterns and its use on the humanoids developed earlier is necessary to get an initial idea of how to start developing the strategy for a walking biped. The aim of this paper is to analyze, design and control the walking of a biped robot. Dimension of each part of robot is established according to the human proportions for robot to have a human-like walk. The torque requirements are established through inverse dynamic analysis applying Newton-Euler recursive equations at the system of lower limb. Offline and online control strategy for the robot are developed using various gait parameters.

Keywords: Gait Patterns, Dynamic Analysis, Control Strategy.

1 Mechanical Design of Robot

The Mechanical Design of the humanoid robot begins by fixing its structural specifications, i.e. height, width etc. The height of the robot is chosen which is most convenient to manufacture and easy to work. For the robot to be able to perform human like walk, an analysis of the human body must be made. The human proportions are given by Leonardo da Vinci's Vitruvian man [1] as shown in Fig. 1.

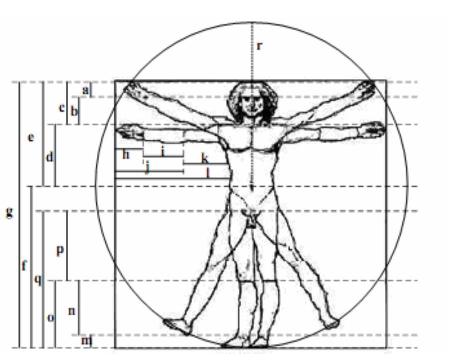

Fig. 1: Da vinci's Vitruvian man [4]

The connection between the proportions of different limbs of the human body and the number phi (φ) is given by:

$$\varphi = 1.618 \text{ and } 1/\varphi = 1/1.618 = 0.618$$

If a circle is drawn with center in the naval of the body, and a radius going from the naval to the feet, this circle encircles the entire body. A square, consisting of the span of the arms and the full body height (which is equal), is drawn. Then the ratio between the radius of the circle and the length of a side in the square equals $\varphi = r/g$.

Based on this, a biped robot is proposed with the following dimensions.
Overall height of the robot = 85 cm.
Foot width = 7.5 cm.
Foot thickness = 3 cm.
Thickness of legs = 6 cm.
Maximum width of robot = 23 cm (from front)
Due to light weight, taking aluminum as the material for the robot (specific weight of Al = 0.027 N/cm^3), the total weight of the robot is equal to 15.2 kg (Approx).
DOF considered for walking = 10 (5 DOF/ lower limb);
DOF for ankle joint = 2, DOF for knee joint = 1,
DOF for hip joint = 2.

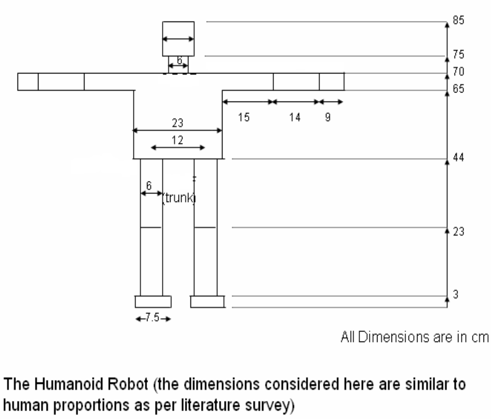

Fig. 2: Dimensions of the Robot

After fixing the dimensions of different parts of robot as stated above, material used to design the robot are chosen depending on the required strength. The material used in this design is aluminium as it ensures both high strength and less weight. Once the physical characteristics of the robot are known, an Inverse dynamic analysis is carried out to find the torque required at each joint. First forward kinematic analysis is done which gives the linear and angular acceleration and then inverse dynamic analysis is carried out. The torque requirement helps in finding the suitable motor to actuate the joints. Here the analysis is carried out for bipedal motion of the robot.

2 Kinematic and Dynamic Analyses

Kinematic analysis is performed using Denavit Hartenberg representation and the forward recursive equations to obtain the relation between angular displacement, velocity, and acceleration in terms of joint and link parameters. Further using Newton-Euler backward recursive equations for dynamic analysis the required torques at different joints are determined.

2.1 KINEMATIC ANALYSIS

The kinematic analysis of the open chain system comprising position, velocity and acceleration analysis, of both translatory and rotary motion, can be divided into the following steps.

• Determine number of body parts and apply local coordinate system to each part.
• Determine the transformation matrix associated with each body part.
• Determine the position of each part in local coordinate system in local coordinates and global coordinates.
• Determine the relative (angular) velocity and acceleration of each joint.
• Determine angular velocity of each body part.
• Determine angular acceleration of each body part.
• Determine the velocity of each body part.
• Determine the acceleration of each body part.

For the present biped motion only rotary joints have been considered.

2.2 The Denavit-Hartenberg Representation

To find translational and rotational relationship between adjacent links, Denavit-Hartenberg representation [2] is used. Through sequential transformations, the end effector expressed in the "leg coordinates" can be transformed and expressed in the "base- coordinate" which makes up the inertial frame of this dynamic system.

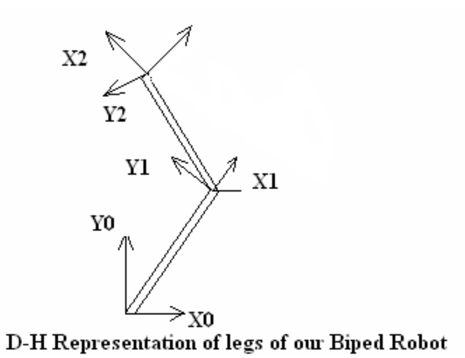

Fig. 3: D-H representation of the legs of the Biped.

Every coordinate frame is determined and established on the basis of three rules:

1. The Z_{i-1} axis lies along the axis of motion of the ith joint.
2. The Xi axis is normal to the Z_{i-1} axis, and pointing away from it.
3. The Yi axis completes the right-handed coordinate system as required.

2.3 DYNAMIC ANALYSIS

Here Newton-Euler formulation is used for finding joint torque. The kinematic information of end effecter starting from base using forward recursive equations is found and later it is used to find the dynamic information (force and torque) at the end effecter. The dynamic information at the end effecter is used to find the dynamic information at base using backward recursive equations. This analysis is being done for the single support phase (SSP).
The variables used in the equations are:

Mi = total mass of link *i*
Ri = position of the center of mass of link *i* from the origin of the base reference frame
Si = position of the center of mass of link *i* from the origin of coordinate system (*Xi*, *Yi*, *Zi*)
*P*i* = the origin of the ith coordinate frame with respect to the (*i-1*)th coordinate system
$V_i = \frac{dR_i}{dt}$, linear velocity of the center of mass of link *i*.
Ai= *dVi/dt*, linear acceleration of the center of mass of link *i*
Fi = total external force exerted on link *i* at the center of mass
Ni = total external moment exerted on link *i* at the center of mass
Ii = Inertia matrix of link *i* about its center of mass with reference to the coordinating system (*Xo*, *Yo*, *Zo*)

fi = force exerted on link *i* by link *i-1* at the coordinate frame (X_{i-1} , Y_{i-1} , Z_{i-1}) to support the link *i* and the link above it.
ni = moment exerted on link I by link *i-1* at the coordinate frame (X_{i-1} , Y_{i-1} , Z_{i-1})

2.4 Backward equations:

i = n, n-1,…..,1
Ni = Ii dWi/dt + Wi x (Ii Wi) (1)
fi = Fi + fi+1 (2)
ni = (ni+1) + P*I x (fi+1) + (P*i + Si) x Fi + Ni (3)

Assumptions:

1. $l_1 = l_2$; The length of lower legs and thighs are almost equal.
2. M_u = 10 kg. ; Mass of the Upper half of Body.
3. M_L = 3.6 Kg ; Mass of the lower half of the Body.
4. ω_1 = (π/6) Rad/sec.
5. ω_2 = (π/3) Rad/sec.
6. ω_3 = (π/2) Rad/sec.

θ_1 = (π/6) + ω_1 t; angular displacement of ankle joint.

$\theta_2 = (\pi/3) - \omega_2 t$, angular displacement of knee joint
$\theta_3 = -\pi/4 + (\pi/2)t$, angular displacement of hip joint and $\ddot{\theta}_1 = \ddot{\theta}_2 = 0$

2.5 Force equation –

For (i=1)

${}_1R_0$ **, general rotation matrix for transforming one coordinate system to another.**

$C_{1,}$ cos (θ_1), similarly $S_{1,}$ sin (θ_1)
C_{12} cos ($\theta_1 + \theta_2$).

$$ {}_1R_0\ f_1 = {}_1R_2\ ({}_2R_0\ f_2) + {}_1R_0\ F_1 \qquad (4)$$

$$ {}_1R_0\ f_1 = {}_1R_2\ ({}_2R_0\ f_2) + M_L\ {}_1R_0\ \overline{a_1} \qquad (5)$$

2.6 Torque Equation –

$$ {}_1R_0\ n_1 = {}_1R_2\ [\,{}_2R_0\ n_2 + ({}_2R_0\ P_1^*)({}_2R_0\ f_2)] + ({}_1R_0\ P_1^* + {}_1R_0\ f_1)({}_1R_0\ F_1) + {}_1R_0\ N_1 \qquad (6)$$

Solving above forward and backward recursive equations we get,

$$T_1 = l\,C_1\,(M_L + M_u)g + gl\,C_1 - l\,S_2/3\,(\dot{\theta}_2 + 2\dot{\theta}_2\,\dot{\theta}_1) - M_L\ gl\ C_3 - l\ C_{123}\ (M_L + M_u) + gl\ C_{123}/3 + l_3/3 + S_3\ \dot{\theta}_1 + gl\,C_{12}/3 \qquad (7)$$

$$T_2 = -\,(M_l\ gl\ C_3 + l\ C_{123}\ (M_L + M_u)g) + l/3\ (S_2\dot{\theta}_1 + g\,C_{12}) \qquad (8)$$

A plot of ankle joint torque T_1 and knee joint torque T_2 with time have been determined using MATLAB and are shown in the Fig. 4 and Fig. 5, respectively. These figures depict the torque over a cycle of walk (SSP).

A cycle of walk can be divided into two phases, viz., SSP, single support phase in which only one foot is in contact with the ground and DSP, double support phase in which both the feet are in contact with the ground.

2.7 Torque Variation with time

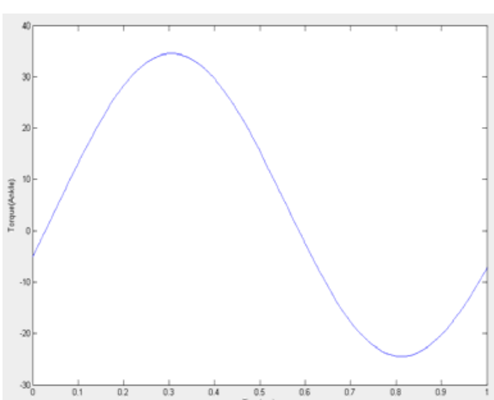

Fig. 4: Variation of ankle torque (N-m) with time (s)

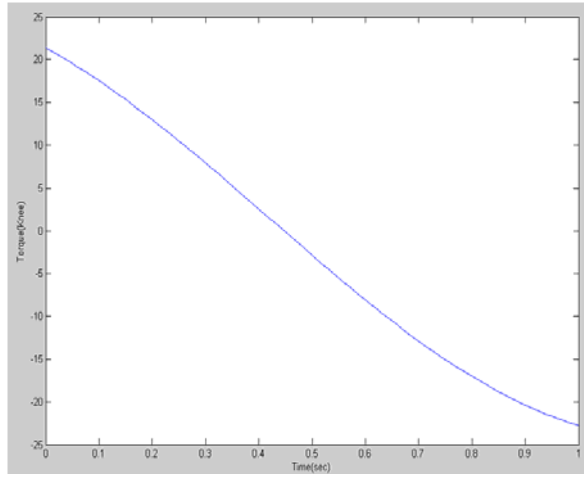

Fig. 5:Variation of knee joint torque (N-m) with time (s)

3 Control Strategy

Here the control strategy is developed for the robot using the forward dynamic analysis. This is carried out to verify whether the loading pattern is valid and if possible improve the precision of the loading pattern. Furthermore it will also verify if the selected power transmission components are adequate. It is started by looking at the geometrical measures used to describe the gait followed by the robot and then implementing those parameters to design offline and online control strategy.

3.1 Desired Gait

In order to completely describe a certain gait, there are also a number of geometrical measures, which must be explained, as shown in figure 6. These measures are self-explanatory. In addition to the measures mentioned in the figure, there is the step height, which is defined as the vertical distance between the floor and the foot in swing phase.

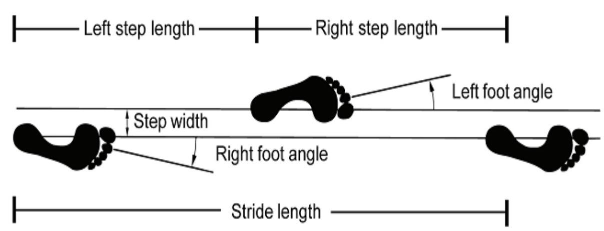

Fig. 6: Geometrical measures of Biped walking

The control strategy is based on two phases:
The offline motion planning concerned establishing a feasible gait pattern and transforming this into join reference trajectories. **The online control** makes the robot follow the predetermined trajectories using PID feedback controllers.

The desired gait is first described by setting the parameters. A gait pattern is then generated, with the necessary reference trajectories in Cartesian coordinated r(ref) which is then transformed by the inverse kinematics to the joint reference trajectories q(ref). Forward dynamic analysis is then initiated and the joint trajectories are followed using a PID controller, which determines the joint torques Mm, based on the position error qerr. The output from the analysis q can then be evaluated.

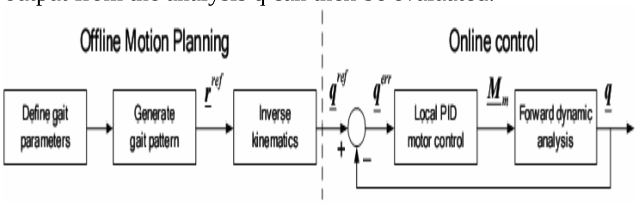

Fig. 7: control strategy

3.2 Offline motion planning

The offline motion planning applied in the control strategy is based on the ZMP approach to determine the ankle trajectories, and an inverted pendulum approach to determine the pelvis trajectories. In the end, inverse kinematics is applied for the transformation of the ankle and pelvis trajectories into joint angular trajectories.
To determine the trajectory for ankle and pelvis centre of mass (CoM), a number of parameters that describes the desired gait must be set. These parameters are listed in Table 1 with the applied values. The values like step length and frequency can be changed to obtain different gait pattern.

Table-1: Offline parameters of Biped Robot

Parameters	**Value Applied**
Step Frequency, *Fs*	1 Hz
Step Time *t* = 1/Fs	1 Sec
SSP/DSP ratio, *r*	0.7
Duration of SSP, *Ts* = rT	0.7 Sec
Duration of DSP, *Td* = (1-r)T	0.3 Sec
Step Width *w*	0.12 m
Step Height *h*	0.02 m
Step Length *L*	0.25 m
Initial Stand Time	0.1 Sec
First step Factor	0.5

Most of the parameters in Table 1 are self explanatory expect the last three. The DSP width is the sideway position of the pelvis, when leaving the DSP. The initial stand time, is the time during which robot remains in the initial position and posture, before commencing the walking. The First Step factor, scales down the step length and height of the first step, in order to initiate the walking smoothly.
The following trajectory need to be determined, and will hence forth be called as the Gait Pattern.

1. Left and Right Ankle trajectories.
2. ZMP trajectory (in the floor plane)
3. CoM trajectory (assumed to be coincident with Pelvis CoM)

3.3 Required Information and ZMP Trajectory

Decent bookkeeping is essential in motion planning, since all the trajectories and generated based on individual phase of walking, the robot goes through. The figure 8, below shows some basic information needed. i.e. phase (left/right or DSP or SSP), Step Number, foothold positions and ZMP trajectory. The one step here is defined as DSP followed by the SSP. All this information is relatively easy to set up as a function of time, based on the gait parameters mentioned in above Table-1. The ZMP trajectory is only set for the SSPs, where ZMP should be located stationary under the legs, During the DSP, ZMP should be moved smoothly from under the trailing foot to under the leading foot.

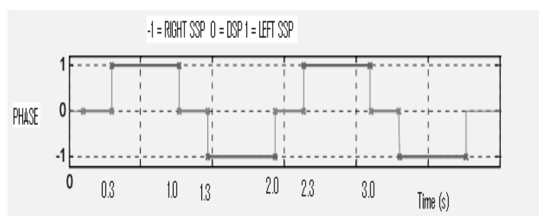

Fig. 8: Walking Phase Diagram.
To ensure a smooth movement of the ZMP, the following cosine functions are applied. They are both given relative to the previous position of the ZMP.

$$X_{zmp} = l/4 - l/4 \cos(\pi\ td/Td) \quad (9)$$
$$Y_{zmp} = (-1)^n w/2 \cos(\pi\ td/Td) \quad n = 0,1,2.. \quad (10)$$

With Boundary Conditions (DSP):
At t = 0 sec, Xzmp (t) = 0 m, Yzmp (t) = .06 m;
At t = 0.3 sec, Xzmp (t) = 0.25m, Yzmp (t) = -.06 m.

The plots shown here show the Xzmp and Yzmp with respect to time. The ZMP trajectory in XY plane can be plotted as shown in Fig 9. During a DSP, the ZMP is always moved from the previous position to the next, determined by the footholds. As seen, the ZMP is moved half a step length forward, and a whole step width sideways.

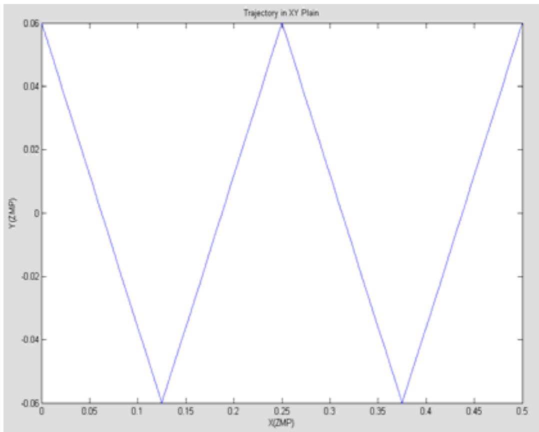

Fig. 9: ZMP trajectory (In meters) in XY plane

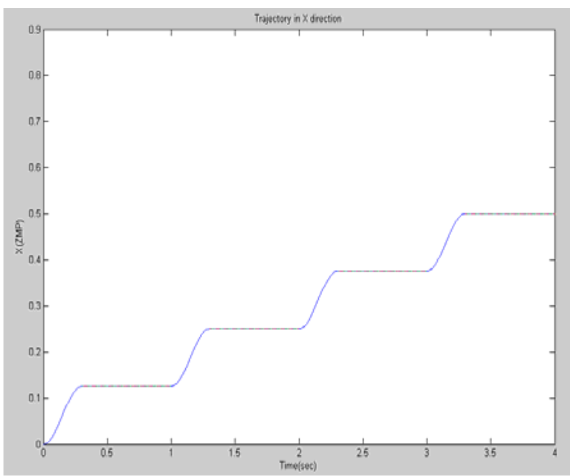

Fig. 10: ZMP trajectory (In meters) in X axis

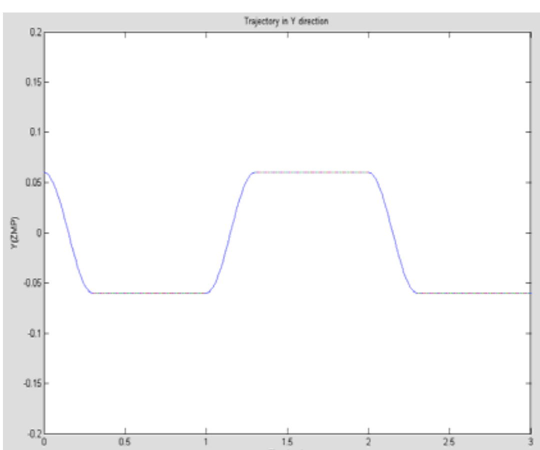

Fig. 11: ZMP trajectory (In meters) in Y axis

3.4 Pelvis Trajectory

To obtain a reference trajectory for the pelvis CoM, the gait is as in the previous section divided in two, namely SSP and DSP. In the SSP, the robot is approximated as a inverted pendulum as illustrated in Figure-12, where the base of the pendulum coincides with the ZMP under the supporting leg (ankle). This is a rather crude approximation, which means that the dynamics of the moving parts e.g. the swing leg, is neglected. This approach is called the 3D-LIPM, for 3D Linear Inverted Pendulum Model.

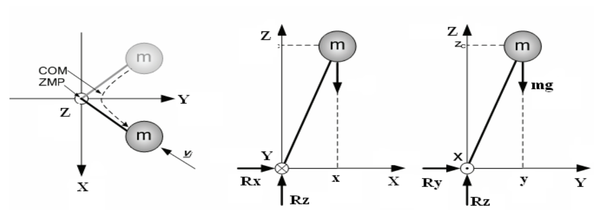

Fig. 12: Mechanism of 3D-LIPM Model.

The equations of motion for the 3D-LIPM can be obtained by force and moment equilibrium, firstly determining the reactions in the origin.

$$\sum fx : Rx = m\ddot{x} \; ; \; \sum fy : Ry = m\ddot{y} \; ;$$

$$\sum fz : Rz = mg$$

$$\sum Mx : Ry\ Zc - R_z\ y = 0$$

$$\Rightarrow m\ddot{y}\ Z_c - mg\ y = 0 \qquad (11)$$

$$\sum M_y : -R_x\ Z_c + R_z\ x = 0$$

$$\Rightarrow -m\ddot{x}\ Z_c + mg\ x = 0 \qquad (12)$$

This can be written into the following two differential equations, which describes the dominant dynamics of the robot. They describe a relationship between the position and acceleration of the CoM which will maintain the balance of the robot.

$$\ddot{x} = \mathbf{g}/Z_c\ x \qquad (13)$$

$$\ddot{y} = = \mathbf{g}/Z_c\ y \qquad (14)$$

3.4.1 Solving 3-D LIMP equations

Because of their simplicity, the equations can be solved analytically by applying the following general solutions. The differential equations above are initial value problems, and their solution is given as

$$X(t) = C_1 e^{\omega t} + C_2 e^{-\omega t} \; ; \qquad (15)$$

$$Y(t) = C_3 e^{\omega t} + C_4 e^{-\omega t} \, . \qquad (16)$$

Where $\omega = \sqrt{g / Zc}$

With boundary conditions,
$X(0) = -0.03125$ m, $X(.3) = +0.03125$ m.
$Y(0) = +0.06$ m, $Y(.3) = -0.06$ m.

Note: the above equations are for SSP. In DSP CoM moves along the ZMP trajectory only.
The constants C1,…C4 depends on the initial values of the pelvis CoM position and velocity. The constants are:

$$C_1 = ½ (x_i + 1/\omega \ \dot{x}_i), \quad (17)$$

$$C_2 = ½ (x_i - 1/\omega \ \dot{x}_i), \quad (18)$$

$$C_3 = ½ (y_i + 1/\omega \ \dot{y}_i), \quad (19)$$

$$C_4 = ½ (y_i - 1/\omega \ \dot{y}_i), \quad (20)$$

Calculating these values gives us the pelvis CoM trajectories in the XY Plane

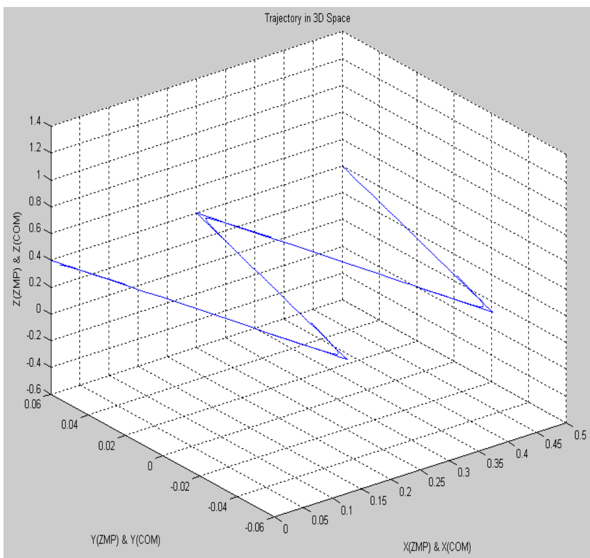

Fig 13: 3 D trajectories of ZMP & CoM (In meters).

In Fig. 13 trajectory of the CoM follows the trajectory of the ZMP in the DSP and resides inside the trajectory of ZMP which is the basis of stable dynamic walk of a biped.
The pelvis CoM trajectory to be followed during SSP and DSP are shown in Fig. 14. Here, 'n' indicates the step number. The motion of the inverted pendulum in step 'n' is given in the x'y' coordinate system. The trajectories and the initial velocities of both ZMP and CoM have been shown in subsequent steps in this figure. The SSP is set to be initiated at a defined time, however, it is also necessary to define the sideway position of the pelvis CoM at this given time. This position is denoted 'b' in Fig. 14 and defines both **Y**i and $\mathbf{Y}_f$. **X**i and $\mathbf{X}_f$ is then determined by the intersection between the line defined by b and the ZMP trajectory. The initial and final position of the pelvis CoM is thereby determined.

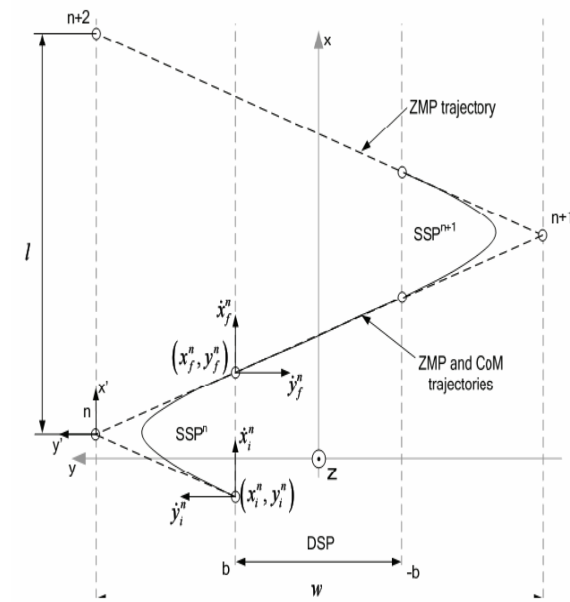

Fig. 14: Explanation of ZMP and CoM trajectories in XY Plane.

3.5 Angular Trajectories

To make the ankles and pelvis CoM follow their respective trajectories, the joint angles are to be determined over time. It is assumed that the only joints that contributes to the motion of the before mentioned body parts, are the joints located below the waist joint. The remaining joints are to maintain their initial positions.

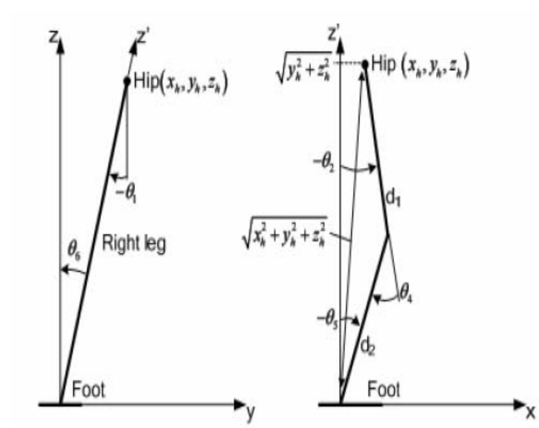

Fig. 15: Angular displacements

Using the trajectory of CoM pelvis and simple trigonometric relations in the two orthogonal planes shown in Fig. 15, the desired angular displacements of the ankle, knee (Fig 16) and hip joint are found for SSP.

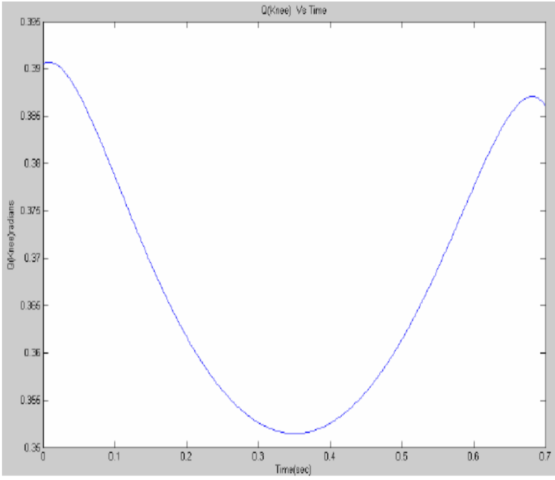

Fig 16: Desired Angular Trajectory (radians) Vs Time(s) for Knee Joint

4 Online Controls

4.1 Dynamic Equations for swing Phase

In the single support phase, one leg is pivoted to the ground while the other is swinging in the forward direction. The dynamic equation of the swinging leg during the swing phase is of the form,

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + g(\theta) = \tau \qquad (21)$$

Using above equation the dynamic model for the biped robot is given by the following equations.

$$\tau_1 = \left[(m_1/3 + m_2)a_1^2 + m_1 a_1 a_2 + (m_2 a_2^2)/3\right]\ddot{q}_1 + \left(\frac{m_2 a_1 a_2 C_2}{2} + \frac{m_2 a_2^2}{3}\right)\ddot{q}_2 - m_2 a_1 a_2 s_2\left(\ddot{q}_1\ddot{q}_2 + \frac{\ddot{q}_1}{2}\right) + g_0\left[\left(\frac{m_1}{2} + m_2\right)a_1 C_1 + \frac{m_2 a_2 C_{12}}{2}\right] + b_1(\dot{q}_1) \qquad (22)$$

$$\tau_2 = \left(\frac{m_2 a_1 a_2 C_2}{2} + \frac{m_2 a_2^2}{3}\right)\ddot{q}1 + \frac{m_2 a_2^2 \ddot{q}_2}{3} + \frac{m_2 a_1 a_2 S_2 \dot{q}_1^2}{2} + \frac{g_0 m_2 a_2 C_{12}}{2} + b_2(\dot{q}_2) \qquad (23)$$

Error (e): E = desired τ - Actual τ .
Actual torque at any time t due to error is given by,

$$\tau(t) = kpe(t) + ki\int e(\tau)d\tau + kD\dot{e}(t) \qquad (24)$$

Using the above dynamic equations of a two-axis planer biped model, the trajectory of the angles calculated from the offline control strategy and the PID error analysis, a code is developed to calculate the error in torque in the joints of lower limb which comes out to be inside a maximum range of 0.02. This range is acceptable for stability.

4.2 Control Algorithm:

Controlling parameter is torque and controlled parameter is the angle (Q) which is used to describe the gait

1. Initially, Q = f(t)
2. Give an initial value of torque
3. find Qactual(t) by solving the equation
   $$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta}) + g(\theta) = \tau$$
4. Error in Q(t) can be given as
   e = Q (desired) – Q (actual)
5. Actual torque at any given time is given by, $\tau(t) = k_p e(t) + k_i\int e(\tau)d\tau + k_D\dot{e}(t)$
6. Value of torque is calculated by using the error in the first step.
7. Then this new torque is used to calculate a new Qactual (t) which is used to calculated the new error.
8. Above steps are repeated in a recursive manner till we get the error in acceptable range for the whole cycle time.

5 Conclusions

In this work biped robot dimensions have been calculated and trajectories for ZMP, CoM, ankle and hip joints were calculated using offline parameters for the biped stability. Also maximum torque requirements for the ankle and knee joint are calculated for selection of proper actuator for these joints of the lower limb. An online PID control scheme was implemented and the error is found to be in the acceptable range. This ensures robot stability during the actual straight walk. This work can be used as a blueprint to design the walking strategy of a biped robot. The walking style generated is more human like than other biped's having a walking strategy of an old man with a back problem.

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