Synthesis of Kinematics Chains with and without Clearance using GA

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Abstract

Multi-bar kinematic chains are used for path, motion and function generation. From kinematics points of view linkages are made of rigid bodies with no clearance at the joints. But actually they are made of flexible materials with tolerance and clearance at the joints and are called flexible linkages. The clearance produces errors in the motion generated by the coupler point. A path generation 4-bar linkage, having clearances at two revolute joints, is considered to investigate the effect of the joint clearance on the coupler path and transmission quality. The joint clearance is modeled as a virtual link, the direction of which with respect to input position is determined by the Lagrange's equation. The GA is used for optimization of link parameters for minimizing the error between desired and actual path due to clearances. Continuous contact between the pin and socket is assumed when clearance is considered.

Keywords: Clearance, Genetic algorithm, Lagrange's equation and Path generation

1 Introduction

When clearance in joints, and deformation and tolerance in links are assumed, the resulting linkages are called flexible linkages [1]. A number of works are reported in journals on flexible linkages as well as rigid linkages. Kunjur and Krishnamurty [2] presented a study on the application of GA technique in the synthesis of linkages. Radovan R., Bulatovic´ A, and Stevan R. Dordevic´ [3] worked on the optimum synthesis of a 4-bar linkage to make the coupler point generate approximately a rectilinear motion. A Grashof 4-bar linkage was considered and a controlled deviation from the given rectilinear path was allowed. Differential Evolution (DE) algorithm that has a similarity with genetic algorithm (GA) was used for optimum synthesis of the linkage. In [4] Dhargupta and Naskar presented a paper on path synthesis of planar multi-bar linkages using DE algorithm. In all these works [2-4] no joint clearance was considered in the linkages.

Many designers have investigated the effects of joint clearance on the performance of linkages. Ting et al [5] have presented an approach to identify the worst position errors due to the joint clearance in linkages and manipulators. Joint clearance was modeled as a small link with length equal to the clearance. A geometrical model was used in their method to assess the output position or direction variation to predict the limit of position uncertainty and to determine the maximum clearance and precision of mechanisms. Tasi and Lai [6] have presented an effective method to analyze the transmission performance of linkages with joint clearances. Equivalent kinematic pairs were used for modeling the motion freedoms originated from the joint clearance. Cabrera et al. [7] have presented methods of solutions for the optimal synthesis of planar mechanisms using GA. In [8] S. Erkaya and I. Uzmay have proposed a model for considering joint clearance as a virtual link and a method to find out the direction of the virtual link. In [9] path generation 4-bar kinematics chains are designed with the objective of minimizing path error using GA for both joint clearance and no clearance. Effect of the joint clearance is investigated also on transmission angle.

Objective of the present work is to design a 4-bar linkage, for the purpose of path generation, having clearances at two revolute joints connecting the driver and the driven links with the coupler. The effects of the joint clearances on path generation and transmission quality are investigated. Comparison is done between linkages, with and without clearances, on the basis of position, velocity and acceleration of the mass-centers of the coupler and the follower. The joint clearance was modeled as a virtual link, the direction of which with respect to input position is determined by the Lagrange's equation. The GA is used for optimization of link parameters and minimizing the error between desired and actual path due to clearance. Here continuous contact between the pin and socket is assumed when clearance is considered. For, if we do not consider continuous contact, the length of clearance vector would have been changed with respect to input position of the driver crank.

2 Joint clearances

Joint clearance r_2 (Fig. 1) is defined as the difference between the radii of bearing and journal, r_B and r_j. If no lubrication is used, the journal can move freely within the bearing until any contact between the two bodies takes place. When the journal impacts on the bearing wall, normal and tangential forces occur. Also, if the friction is negligible, the direction of joint clearance vector coincides with the direction of normal force at the contact point. When the continuous contact mode between journal and bearing is considered, the clearance may be modeled as a vector which corresponds to massless virtual link with length equal to joint clearance. The equivalent clearance vector can be defined in the form,

$$r_2 = r_B - r_j \qquad (1)$$

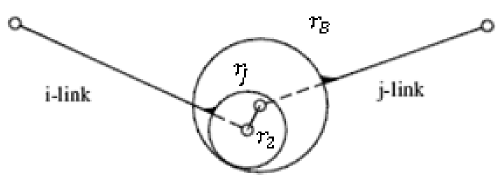


Fig. 1 Equivalent clearance link.

3 Synthesis of 4-bar linkages with two clearances for path generation

Dimensional synthesis of path generation planer linkages leads to determination of link dimensions on the basis of desired path and minimization of structural error subjected to a set of size and geometric constraints based on Grashof's rules. Fig. 2 shows the 4-bar chain while Fig. 3 is its vector representation with two clearances. Link parameters of the 4-bar chain are given in Table 1.

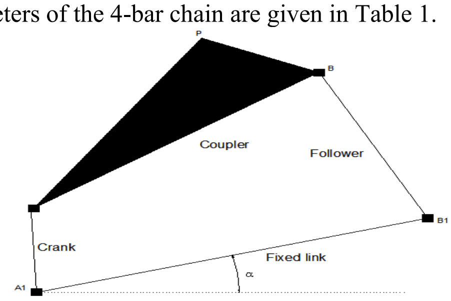


Fig. 2 4-bar linkage

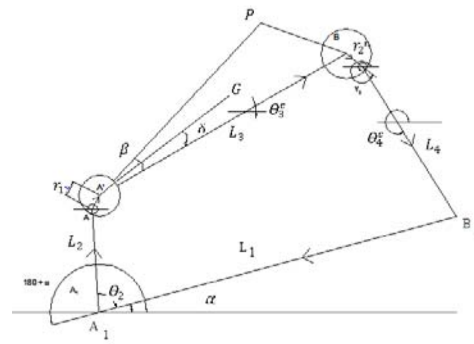


Fig. 3 Vector diagram of 4-bar linkage with clearances.

Table 1. Parametric values of the 4-bar linkage

Link	Length(mm)
Fixed link (1)	107.5
Crank (2)	31.3
Coupler (3)	121.0
Follower (4)	91.9

From the loop closure vector relation

$$L_1 e^{i(\alpha+180)} + L_2 e^{i(\alpha+\theta_2)} + r_1 e^{i\gamma_1} + L_3 e^{i\theta_3} + r_2 e^{i\gamma_1} + L_4 e^{i\theta_4} = 0 \qquad (2)$$

This leads to

$$\theta_3^c = 2\arctan\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \qquad (3)$$

$$\theta_4^c = \cos^{-1}\left[\frac{L_1\cos\alpha - L_2\cos(\theta_2+\alpha) - r_1\cos\gamma_1 - L_3\cos\theta_3^c - r_2\cos\gamma_2}{L_4}\right] \qquad (4)$$

Where the superscript c denotes the value with clearance and the terms A, B, C are (with joint clearances):

$$A = L_1^2 + L_2^2 + r_1^2 + L_3^2 + r_2^2 - L_4^2 - 2L_1L_2\cos(2\alpha+\theta)$$
$$+2r_2r_1\cos(\gamma_1-\gamma_2) - 2L_1r_1\cos(\alpha+\gamma_1) - 2L_1r_2\cos(\alpha+\gamma_2)$$
$$+2L_2r_1\cos(\alpha+\theta_2-\gamma_1) + 2L_2r_2\cos(\alpha+\theta_2-\gamma_2) -$$
$$2r_1L_3\cos\gamma_1 - 2L_3r_2\cos\gamma_2 + 2L_1L_3\cos\alpha - 2L_2L_3r\cos(\alpha+\theta_2),$$
$$B = 4(r_1L_3\sin\gamma_1 + L_2L_3\sin\gamma_2 + L_1L_3\sin\alpha + L_2L_3\sin(\alpha+\theta_2)),$$
$$C = L_1^2 + L_2^2 + r_1^2 + L_3^2 + r_2^2 - L_4^2 - 2L_1L_2\cos(2\alpha+\theta)$$
$$+2r_2r_1\cos(\gamma_1-\gamma_2) - 2L_1r_1\cos(\alpha+\gamma_1) - 2L_1r_2\cos(\alpha+\gamma_2)$$
$$+2L_2r_1\cos(\alpha+\theta_2-\gamma_1) + 2L_2r_2\cos(\alpha+\theta_2-\gamma_2) +$$
$$2r_1L_3\cos\gamma_1 + 2L_3r_2\cos\gamma_2 - 2L_1L_3\cos\alpha - 2L_2L_3\cos(\alpha+\theta_2)$$

Similarly, θ_3 and θ_4, for no joint clearance can be expressed in the following forms as functions of θ_2 only,

$$\theta_3 = 2\arctan\left[\frac{-B \pm \sqrt{B^2 - 4AC}}{2A}\right] \qquad (5)$$

$$\theta_4 = \cos^{-1}\left[\frac{L_1\cos\alpha - L_2\cos(\theta_2+\alpha) - r_2\cos\gamma_2}{L_4}\right] \qquad (6)$$

$$A = L_1^2 + L_2^2 + r_1^2 + L_3^2 + r_2^2 - L_4^2 - 2L_1L_2\cos(2\alpha+\theta)$$

$$+2L_1L_3\cos\alpha - 2L_1L_3\cos(\alpha+\theta_2),$$

$$B = 4(L_1L_2\sin\alpha + L_2L_3\sin(\alpha+\theta_2)),$$

$$C = L_1^2 + L_2^2 + r_1^2 + L_3^2 + r_2^2 - L_4^2 - 2L_1L_2\cos(2\alpha+\theta)$$

$$-2L_1L_3\cos\alpha + 2L_2L_3\cos(\alpha+\theta_2)$$

As shown in Fig. 3 the coordinates of the coupler point P relative to the crank pivot A_1 is given with joint clearance,

$$P_x^c = L_2\cos(\theta_2+\alpha) + r_1\cos\gamma_2 + A'P\cos(\theta_3^c+\beta) \quad (7)$$

$$P_y^c = L_2\sin(\theta_2+\alpha) + r_1\sin\gamma_2 + A'P\sin(\theta_3^c+\beta) \quad (8)$$

where P_x^c, P_y^c denote the x and y coordinates of the coupler path with joint clearances.

The coordinates of the coupler point P relative to the crank pivot A_1 is given for no joint clearance

$$P_x = L_2\cos(\theta_2+\alpha) + A'P\cos(\theta_3+\beta) \quad (9)$$

$$P_y = L_2\sin(\theta_2+\alpha) + A'P\sin(\theta_3+\beta) \quad (10)$$

where P_x, P_y denote the x and y coordinates for the coupler path for no joint clearance .

Kinematic analysis of 4-bar mechanism requires determination of position and corresponding velocities and accelerations of mass center of moving links. For joint clearance these parameters are derived from the vector representation of the linkage shown in Fig. 3.

Due to motion transmission from crank to follower, joint clearance between crank and coupler has important role on coupler path generation. With reference to the pivot A_1, the mass center positions for moving links with clearances are:

$$x_{G2}^c = A_0G_2\cos(\theta_2+\alpha) \quad (11)$$

$$y_{G2}^c = A_0G_2\sin(\theta_2+\alpha) \quad (12)$$

$$x_{G3}^c = L_2\cos(\theta_2+\alpha) + r_1\cos\gamma_1 + A'G_3\cos(\theta_3^c+\delta) \quad (13)$$

$$y_{G3}^c = L_2\sin(\theta_2+\alpha) + r_1\sin\gamma_1 + A'G_3\cos(\theta_3^c+\delta) \quad (14)$$

$$x_{G4}^c = L_2\cos(\theta_2+\alpha) + r_1\cos\gamma_1 + L_3\cos\theta_3^c + BG_4\cos\theta_4^c \quad (15)$$

$$y_{G4}^c = L_2\sin(\theta_2+\alpha) + r_1\sin\gamma_1 + L_3\sin\theta_3^c + BG_4\sin\theta_4^c \quad (16)$$

The mass center positions of the moving links without joint clearance are:

$$x_{G2} = A_0G_2\cos(\theta_2+\alpha) \quad (17)$$

$$y_{G2} = A_0G_2\sin(\theta_2+\alpha) \quad (18)$$

$$x_{G3} = L_2\cos(\theta_2+\alpha) + AG_3\cos(\theta_3+\delta) \quad (19)$$

$$y_{G3} = L_2\sin(\theta_2+\alpha) + AG_3\sin(\theta_3+\delta) \quad (20)$$

$$x_{G4} = L_2\cos(\theta_2+\alpha) + L_3\cos\theta_3 + BG_4\cos\theta_4 \quad (21)$$

$$y_{G4} = L_2\sin(\theta_2+\alpha) + L_3\sin\theta_3 + BG_4\sin\theta_4 \quad (22)$$

4 Transmission Angle

Transmission angle is an important parameter by which the quality of motion transmission in a linkage is judged. It helps in deciding the "best" among a family of possible linkages for most effective force transmission. When transmission angle deviates much from 90°, linkages exhibit poor operational characteristics like noise and jerk at high speed application.

Transmission angle without and with clearance are:

$$\mu = \arccos\left[\frac{L_1^2+L_2^2-L_3^2-L_4^2-2L_1L_2\cos(\theta_2+\alpha)}{-2L_3L_4}\right] \quad (23)$$

$$\mu^c = \arccos\left[\frac{r_2^2-OB^2-OB'^2-2L_1L_2\cos(\theta_2+\alpha)}{-2OB.OB'}\right] \quad (24)$$

$$OB' = \frac{L_3r_2}{A'B_0-r_2} \quad (25)$$

$$OB = \frac{L_4r_2}{A'B_0-r_2} \quad (26)$$

5 Lagrange's equation

If links are assumed to be rigid, the direction of the clearance vector can be derived by using Lagrange's equation as given below

$$\frac{d}{dt}\left(\frac{\partial t}{\partial\gamma_2}\right) - \frac{\partial T}{\partial\gamma_2} + \frac{\partial U}{\partial\gamma_2} + \frac{\partial DF}{\partial\gamma_2} = 0 \quad (27)$$

where T, U, and DF denote the kinetic energy, potential energy, and dissipation energy respectively, which are:

$$T = \frac{1}{2}\sum_{i=0}^{4} I_i(\dot\theta_i^c)^2 + \frac{1}{2}\sum_{i=0}^{4} m_i\left[(\dot x_{Gi}^c)^2 + (\dot y_{Gi}^c)^2\right] \quad (28)$$

$$U = \frac{1}{2}\sum_{i=0}^{4} m_i g y_{Gi}^c \quad (29)$$

$$DF = \frac{1}{2}\sum_{i=0}^{4} C\theta_i\left(\theta_i^c\right)^2 + \frac{1}{2}C_{\gamma 2}\dot\gamma^2 \quad (30)$$

where i is the number of link. If the above terms are substituted in Eq. (27), Lagrange's equation can be derived in the following form

$$\sum_{i=2}^{4}\left[I_i\ddot\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + m_i\left(\ddot x_{Gi}^c\frac{\partial x_{Gi}^c}{\partial\gamma_2} + \ddot y_{Gi}^c\frac{\partial y_{Gi}^c}{\partial\gamma_2}\right) + gm_i\frac{\partial y_{Gi}^c}{\partial\gamma_2} + C_{\theta i}\dot\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + C_{\gamma 2}\dot\gamma_2\right] = 0 \quad (31)$$

6 Optimization Procedure for Path Generation by Linkage with Joint Clearance

Two objective functions (*OF*) are defined, first, to determine the direction of the joint clearance and second, to minimize the error between the desired and actual position of point P. As seen from Eq. (31), the equation of motion has nonlinear character. GA approach is applied to solve this equation for determining the direction of joint clearance as a function of position variable of input link. This equation is considered as first *OF*, and is expressed in the following form:

Minimize

$$F_1(X) = f\left(\sum_{i=2}^{4}\left[I_i\ddot\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + m_i\left(\ddot x_{Gi}^c\frac{\partial x_{Gi}^c}{\partial\gamma_2} + \ddot y_{Gi}^c\frac{\partial y_{Gi}^c}{\partial\gamma_2}\right) + gm_i\frac{\partial y_{Gi}^c}{\partial\gamma_2} + C_{\theta i}\dot\theta_i^c\frac{\partial\theta_i^c}{\partial\gamma_2} + C_{\gamma_2}X\right]\right) \quad (27)$$

subject to

$$h_j(X) = 0, \quad (28)$$

$$x_l \le x_j \le x_u, \quad (29)$$

$$x_j \in= X \quad (30)$$

where $h_j(x)$ are equality constraints which depend on parametric relations between direction of joint clearance and input variable; X is a vector comprising the design variables corresponding to direction, velocity and acceleration of joint clearance, i.e., $\gamma_2, \dot{\gamma}_2, \ddot{\gamma}_2$.

In path generation problems, the coupler point has to track a given path with minimum error. *OF* is a measure of the error between the desired and actual paths, and is usually expressed as the sum of the squares of the errors at each point of the path [10]. To minimize the error between desired and actual positions of point P, the second *OF* is formulated as:

Minimize

$$F_2(X)=\sqrt{\frac{1}{N}\left(\sum_{j=1}^{N}\left[(P^d_{xn}-P^g_{xn})^2+(P^d_{yn}-P^g_{yn})^2\right]\right)} \quad (31)$$

subject to $g_k(x)\le 0,$ (32)

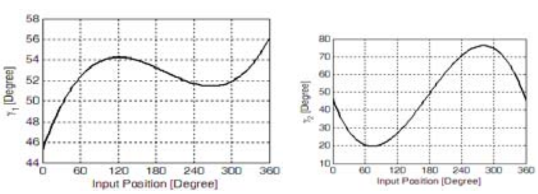
$$x_l \le x_j \le x_u \quad (33)$$

where *d* stands for desired and *g* stands for generated point. The inequality constraints $(g_k(x))$ come of Grashof's rule. Design variables for the second *OF* are:

$$X = \left[L_1 L_2 L_3 L_4 \beta A'P\right] \quad (34)$$

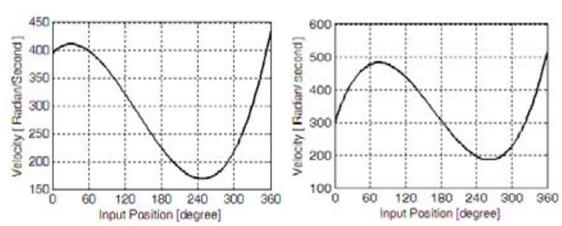
6 Result

In the paper, a theoretical model is used for the case of two clearances in a 4-bar mechanism with rigid links, running at a speed of 600 rpm. The determination of the direction variable of the joint clearance and the optimum link parameters $(L_1 L_2 L_3 L_4 \beta A'P)$ in the model mechanism with joint clearance is realized by using GA on MATLAB software. Therefore, the variation of the direction of joint clearance with respect to the input variable, which is obtained from the first *OF* using GA, is outlined in Figs. 4(a), 4(b), and 4(c).



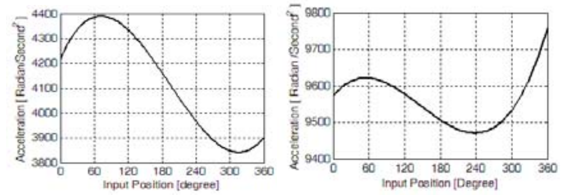
Clearance between the crank and coupler *Clearance between the coupler and follower*

Fig. 4(a) Direction of joint clearance with respect to input link



Clearance between crank and coupler *Clearance between the coupler and follower*

Fig. 4(b) Velocity of joint clearance with respect to input link



Clearance between the crank and coupler *Clearance between the coupler and follwer*

Fig. 4(c) Acceleration of the joint clearance with respects to input link

Table. 2 Optimized design variables for different joint clearance values

Design variables	Original values	$r_2 = 1.0\ mm$	$r_2 = 2.0\ mm$
$L_1\ (mm)$	107.5	106.94	106.86
$L_2\ (mm)$	31.3	31.23	31.19
$L_3\ (mm)$	121	122.11	121.9
$L_4\ (mm)$	91.9	90.63	90.51
β (degree)	5.99	5.79	5.88
A'P (mm)	169.9	169.4	169.2

If the kinematics Eqs. (7), (8), (9) and (10) are solved for two different clearance values, the obtained path configurations for desired (without clearance), actual clearance (with clearance) and optimized (with and without clearance) linkages are outlined in Fig. 5 and 6, respectively. As shown in these figures, the path error reaches the largest value when the mechanism goes into dead-center positions.

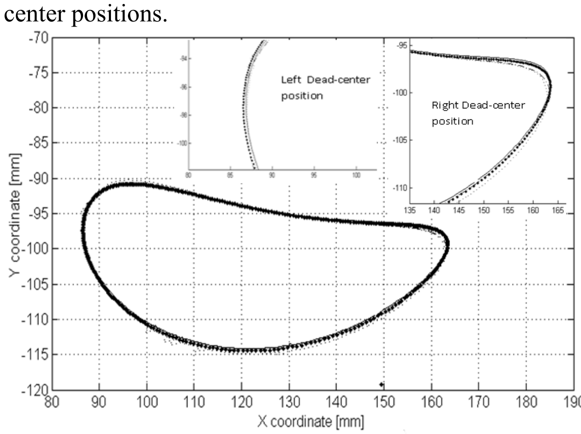


Fig. 5: (…) Desired curve (Without clearance), (—) Optimum curve (without clearance), (_ _ _) Actual curve (with clearance), (••••••) Optimum curve (with clearance) path configurations for the joint clearance of 1 mm.

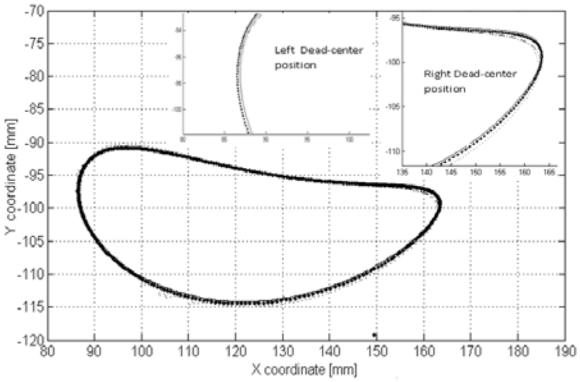
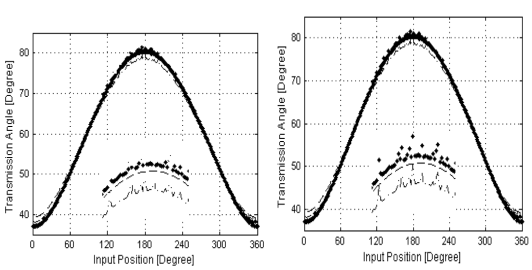


Fig. 6: (…) Desired curve (without clearance), (—) Optimum curve (without clearance), (_ _ _) Actual curve (with clearance), (••••••) Optimum curve (with clearance) path configurations for the joint clearance of 2 mm.

When the values of error in the x and y directions of each linkage are evaluated separately, it is clearly seen that path errors for the joint clearance of 1 mm decrease by 63.38% and 61.45% in the above directions, respectively. For 2 mm clearance, above errors decreases by 62.39% and 61.28%.

7 Transmission Angle

The transmission angle under two different clearance conditions during the whole motion cycle of the mechanism is shown in Fig.7. As seen form figures, the deviation of the transmission angle from the desired value is larger when the value of joint clearance increases. Also, it can be clearly seen that the highest deviation of the transmission angle from the desired value occurs at the extreme position. Compared between the difference between desired and actual transmission angles, and also the deviation of the desired transmission angle from the optimized ones, as shown in Fig.7**,** it is seen that the errors in the latter case are smaller than those of the former. These errors approximately decrease by 54.45% and 39.15% for 1 mm and 2 mm clearances, respectively. Dynamic parameters of the linkage used in Lagrange's equation are given in Table 3.



clearance (1mm) clearance (2mm)

(….) Actual,μ, (_ _) Desired μ , (—) Optimized μ

Fig. 7: Variation of transmission angle

In the kinematics analysis of the 4-bar linkage, it is necessary to determine the position of mass center for moving links and then their corresponding velocities and accelerations. So in case of joint clearance these positions are derived from the vector representation of the linkage in Fig. 2.

Time derivates of the mass center position for moving links yield the mass center velocities and accelerations, with respect to direction of joint clearance are given below

$$\dot{x}^c_{Gi} = \omega_2\left[\frac{\partial x^c_{Gi}}{\partial\theta_2}\right] + \dot{\gamma}_1\left[\frac{\partial x^c_{Gi}}{\partial\gamma_1}\right] + \dot{\gamma}_2\left[\frac{\partial x^c_{Gi}}{\partial\gamma_2}\right] \quad (35)$$

$$\dot{y}^c_{Gi} = \omega_2\left[\frac{\partial y^c_{Gi}}{\partial\theta_2}\right] + \dot{\gamma}_1\left[\frac{\partial y^c_{Gi}}{\partial\gamma_1}\right] + \dot{\gamma}_2\left[\frac{\partial y^c_{Gi}}{\partial\gamma_2}\right] \quad (36)$$

$$\ddot{x}^c_{Gi} = \alpha_2\left[\frac{\partial x^c_{Gi}}{\partial\theta_2}\right] + \omega_2^2\left[\frac{\partial^2 x^c_{Gi}}{\partial\theta_2^2}\right] + 2\omega_2\sum_{j=1}^{2}\dot{\gamma}_j\left[\frac{\partial^2 x^c_{Gi}}{\partial\theta_2\partial\gamma_j}\right] + \sum_{j=1}^{2}\ddot{\gamma}_j\left[\frac{\partial x^c_{Gi}}{\partial\gamma_j}\right] + \sum_{j=1}^{2}\ddot{\gamma}_j^2\left[\frac{\partial^2 x^c_{Gi}}{\partial\gamma_j^2}\right] + \sum_{j=1}^{2}\sum_{k=1}^{2}\left[\frac{\partial^2 x^c_{Gi}}{\partial\gamma_j\partial\gamma_k}\right] \quad (37)$$

for $k \neq j$

$$\ddot{y}^c_{Gi} = \alpha_2\left[\frac{\partial y^c_{Gi}}{\partial\theta_2}\right] + \omega_2^2\left[\frac{\partial^2 y^c_{Gi}}{\partial\theta_2^2}\right] + 2\omega_2\sum_{j=1}^{2}\dot{\gamma}_j\left[\frac{\partial^2 y^c_{Gi}}{\partial\theta_2\partial\gamma_j}\right] + \sum_{j=1}^{2}\ddot{\gamma}_j\left[\frac{\partial y^c_{Gi}}{\partial\gamma_j}\right] + \sum_{j=1}^{2}\ddot{\gamma}_j^2\left[\frac{\partial^2 y^c_{Gi}}{\partial\gamma_j^2}\right] + \sum_{j=1}^{2}\sum_{k=1}^{2}\left[\frac{\partial^2 y^c_{Gi}}{\partial\gamma_j\partial\gamma_k}\right] \quad (38)$$

for $k \neq j$

Time derivates of the mass center position for moving links yield the mass center velocities and accelerations, without clearance are given below

$$\dot{x}_{Gi} = \omega_2\left[\frac{\partial x_{Gi}}{\partial\theta_2}\right] \quad (39)$$

$$\dot{y}_{Gi} = \omega_2\left[\frac{\partial y_{Gi}}{\partial\theta_2}\right] \quad (40)$$

$$\ddot{x}_{Gi} = \omega_2^2\left[\frac{\partial^2 x_{Gi}}{\partial\theta_2^2}\right] \quad (41)$$

$$\ddot{y}_{Gi} = \omega_2^2\left[\frac{\partial^2 y_{Gi}}{\partial\theta_2^2}\right] \quad (42)$$

Where ω_2 and α_2 denote the angular velocity and acceleration of the input link respectively, i denote the moving link number and j denote the joint clearance numbers.

Table. 3 Dynamic parameters for 4-bar linkage

Parameters	Descriptions	Values
I_2	Moment of inertia of the crank	$5.12 \times 10^{-4} kg\ m^2$
I_3	Moment of inertia of the coupler	$8.85 \times 10^{-3} kg\ m^2$
I_4	Moment of inertia of the follower	$1.58 \times 10^{-3} kg\ m^2$
m_2	Mass of the crank	$0.121 kg$

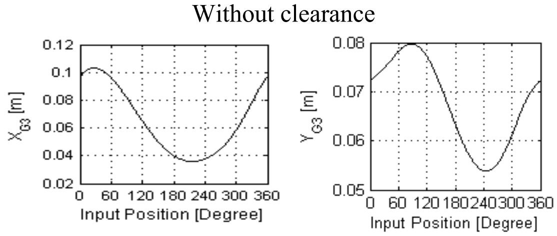
m_3	Mass of the coupler	$1.048kg$
m_4	Mass of the follower	$0.071kg$
g	Acceleration of the gravity	$9.81m/s^2$
ω_2	Angular velocity of input link	$62.83\ rad/s$
$C_{\theta i}$	Damping coefficient [11]	$0.2 \times 10^{-6} kg\ m\ s/$
C_{V_2}	Damping coefficient [11]	$0.2 \times 10^{-6} kg\ m\ s/$

Kinematics equation of coupler mass center with respect to crank pivot A_1comprises without clearance, therefore, positions, velocities and accelerations of this other.

Position analysis of the coupler mass center includes the joint clearance effects of the coupler and follower joint.

In Fig. 8 the deviations between desired and actual positions of the coupler mass center for X and Y directions. In X-direction these deviations are approximately limited between ± 1.1 mm and Y-direction this deviation are bigger (1.2 mm) than X-direction

Without clearance



With clearance

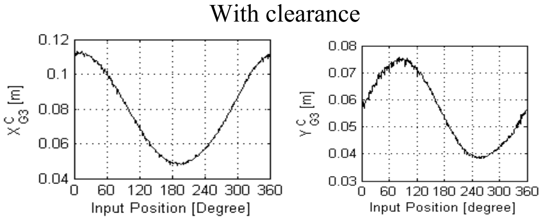
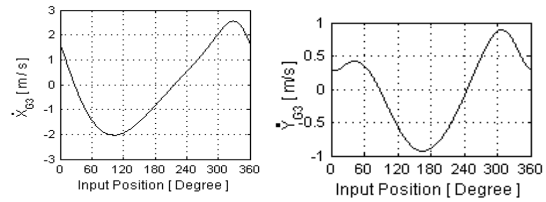


Fig..8: Position analysis of coupler mass center.

Velocity analysis of the coupler mass center for two cases is shown in Fig. 9. The deviation between the desired and actual velocities of the coupler mass center for X-direction are bigger in the beginning of motion compared to Y-direction.

Without clearance



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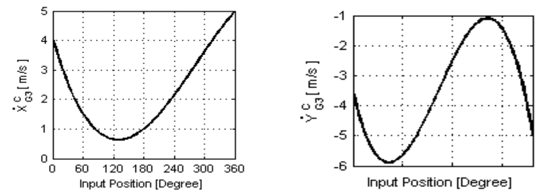
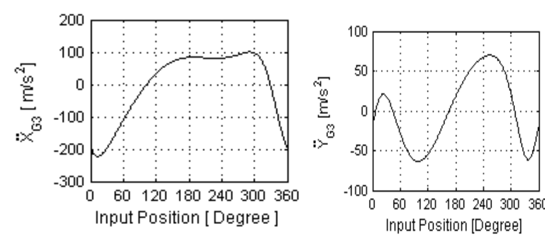


Fig. 9: Velocity analysis of coupler mass center.

Acceleration analysis of coupler mass center is shown in Fig. 10. As seen from figure, there are large deviations from the desired acceleration curve for each direction. Especially, deviation in the Y-direction is bigger than that of X-direction. It can be clearly seen that, these deviations occur in the beginning of the motion that is between 0 and 75° of input variable.

Without clearance



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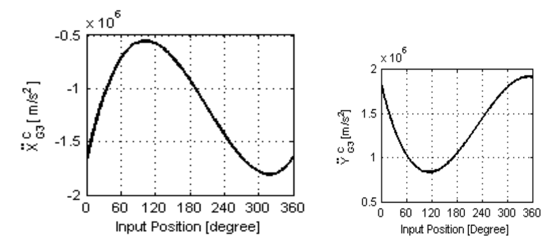
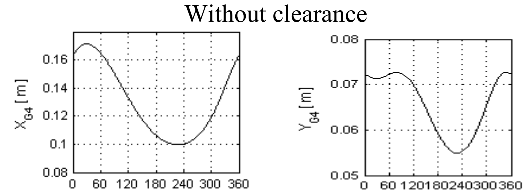


Fig. 10:.Acceleration Velocity analysis of coupler mass center.

In the Fig. 11 the deviations between desired and actual positions of the follower mass center for X and Y-directions. In X-direction these deviations are approximately limited between ± 1.7 mm and in Y-direction these deviations are bigger than those in X-direction.

Without clearance



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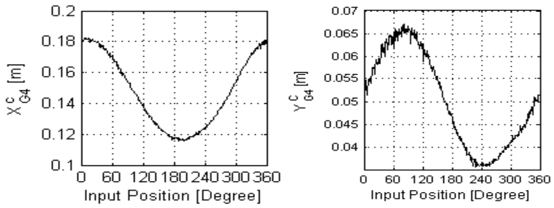
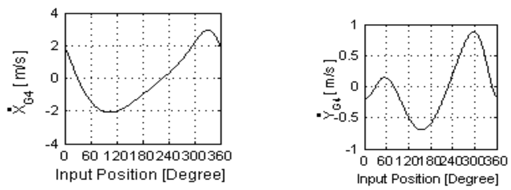


Fig. 11: Position analysis of follower mass center.

Velocity analysis of the follower for two cases is shown in Fig. 12. It is clearly seen during the whole motion cycle that, there are deviations between desired and actual velocities of the follower mass center. The deviation varies in X-direction from 0 and 360° of input variable.

Without clearance



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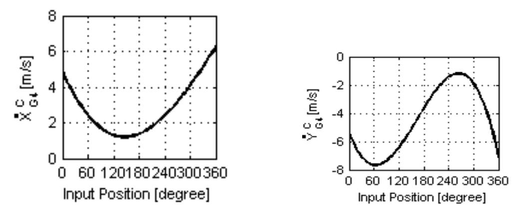
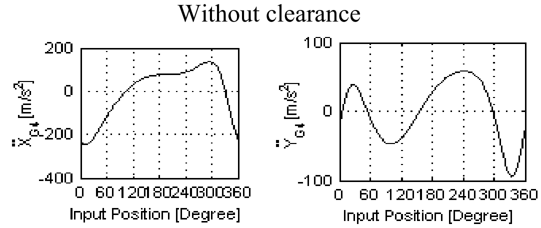


Fig. 12: Velocity analysis of follower mass center.

Acceleration analysis of the follower mass center for two cases is shown in Fig. 13**.** As seen, there are excessive deviations from the desired value in the beginning of the motion. Especially, these deviations lead to sudden changes of direction of inertial forces and these situations result in reducing the dynamic efficiency of the mechanism.

Without clearance



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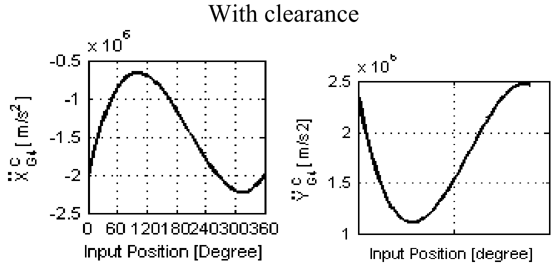

Fig. 13: Acceleration analysis of follower mass center.

8 Conclusion

Directions of virtual links in linkages with clearances are determined. Transmission angles for both the cases - with and without clearances - are compared. It is found that in case of joint with clearances, the deviation of optimized path from actual path is much higher than that when no-clearance is considered. Also if clearances at two joints are taken into account, the path error of the coupler point and deviation of the transmission angle become higher compared to joints with no or single clearance. The position, velocity and acceleration of the mass center position of the coupler and follower are investigated.

9 Further scope of work

The method can be extended for clearance in any number of joints of a constrained multi-bar linkages with any number of links.

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