Estimation of Critical Section and Bending Stress Analysis for Asymmetric Spur Gear Tooth

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Abstract

Gearing is one of the most critical components in mechanical power transmission systems. Transmission error is considered to be one of the main contributors to noise and vibration in a gear set. Transmission error in the gear leads to backlash, undercut and interference. These defects can be eliminated by increasing the pressure angle. Backlash and interference can be avoided by Increase addendum of mating gear; another way of increasing the load capacity of transmissions is to modify the involute geometry. This has been a standard practice in sophisticated gear design for many years. The nomenclature describing these types of gear modifications can be quite confusing with reference to addendum modification or profile shift, etc.

An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth. This is because two profiles of a gear tooth are functionally different for most of the gear drives. The workload on one side of profile is significantly higher than the other side of the gear. An asymmetric spur gear drive means that larger and smaller pressure angles are applied for the driving and coast sides.

The main objective of this paper is to generate asymmetric spur gear tooth profile for different pressure angles on drive and coast sides and estimate the critical section using C- programming. Bending stress analysis has been performed using finite element analysis using ANSYS software .Comparison of bending stress analysis has been performed for symmetric and asymmetric spur gear tooth at critical section.

Keywords: Asymmetric spur gear tooth, pressure angle, critical section, FEM.

1 Introduction

Sufficiency in bending load carrying capacity is a serious problem, as regards the carburized or surface quality improved gears with very high surface fatigue strength, such as plastic and sintered gears. There are several ways to solve the problem such as heat treatments, improving tooth fillet surface quality, and using a larger radius of cutter's tip corner [1]

Another way of increasing the load capacity of transmissions is to modify the involute geometry. This has been a standard practice in sophisticated gear design for many years. The nomenclature describing these types of gear modifications can be quite confusing with reference to addendum modification profile shift, etc. An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth [2]

The aim of asymmetric tooth is to improve the performance of gears such as increasing the load capacity or reducing noise and vibration. Application of asymmetric tooth side surfaces is able to increase the load capacity and durability for the drive tooth side [3]

The tooth form has left–right symmetry in the involute cylindrical gear, and the same performance can be obtained at forward and backward rotation. However, both the forward and backward rotations are not always expected in the practically used gear units for power transmission [1]

Therefore, two sides of the gear tooth are functionally different for most gears. Even if one side (drive side) is significantly loaded for longer periods, the opposite side (coast side) is unloaded or slightly loaded for short duration only [2, 3].

In the literature, there are two different applications for involute gears with asymmetric teeth. The difference between them is the selection of pressure angles for the drive side and coast side. Since the load capacity of the involute gear mainly depends on the pressure angle, the determination of pressure angles is very important.

In this study, asymmetric gears with greater drive side pressure angle than coast side pressure angle are taken into consideration. The purpose of this study is to determine critical section thickness and bending stress distribution for symmetric and asymmetric spur gear drives. Hence, a computer program is used to generate gear tooth profile, estimate bending stress and critical section, which is different from previous studies, which considers pressure angle on the drive side, coast side, module, backup ratio, profile shift, tooth number and contact ratio.

2 Asymmetric spur gear teeth

The two profiles (sides) of a gear tooth are functionally different for many gears. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of the asymmetric tooth shape reflects this functional difference.

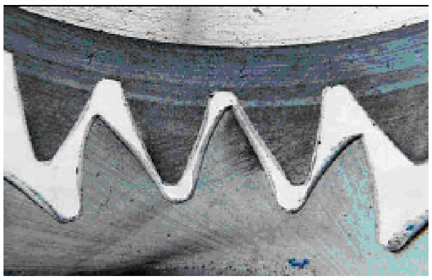

Fig. 1: Asymmetric spur gear.

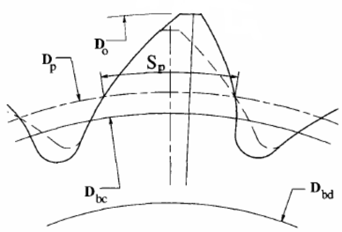

Fig. 2: Asymmetric spur gear with different base circles.

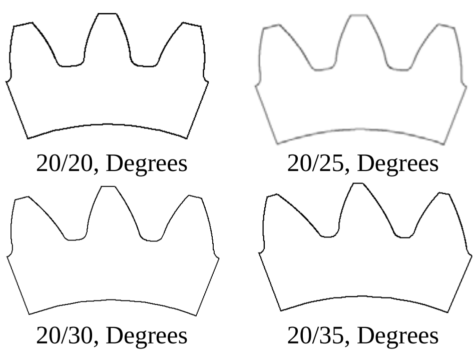

Fig. 3: Tooth forms in different pressure angle combinations

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile. The opposite profile is typically unloaded or lightly loaded during relatively short work periods. The degree of asymmetry and drive profile selection for these gears depends on the application.

The difference between symmetric and asymmetric tooth is defined by two involutes of two different base circles D_{bd} and D_{bc}. The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter D_p that should be bigger than the largest base diameter.

Asymmetric gears simultaneously allow an increase in the transverse contact ratio and operating pressure angle beyond the conventional gear limits. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles by changing the coast side profiles. This provides higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

Tooth forms in different pressure angle combinations on coast (right side) and drive side (right side) as shown in fig.3.

3 Profile generation and Estimation of critical section using C-programme

Critical section is one of the important gear tooth parameter which is to be evaluated for bending stress calculations. There are many methods for the determination of the critical section parameter for nonstandard, external gear teeth, considering tool offsets, center distance modifications and backlash.

Table-1: Gear tooth parameters

Sl.No.	Description	Value
1	Pressure angle, Coast side	20^0 fixed
2	Pressure angle, Drive side	20^0-35^0 increment by 1^0
3	Profile shift factor	0,0
4	Number of teeth	25 and 47
5	Module	4 mm
6	Back up ratio	1.2

In this paper we generate key points for profile generation and estimate the critical section using gear tooth parameters given in table-1.Results obtained will have good agreement with the published one.

$$\bar{r}(\theta) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \quad (5)$$

$$X(\theta) = N\frac{M_n}{2}\left\{\sin\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\cos(\phi + \theta)\right\} \quad (6)$$

$$Y(\theta) = N\frac{M_n}{2}\left\{\cos\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\sin(\phi + \theta)\right\} \quad (7)$$

$$\theta_{min} \le \theta \le \theta_{max} \quad (8)$$

$$\theta_{min} = \frac{2}{N}\left[U + (V + X)\cot\phi\right] \quad (9)$$

$$\theta_{max} = \frac{1}{N\cos\phi} \times \sqrt{(2+N+2X)^2 - N(\cos\phi)^2 - \left(1+\frac{2X}{n}\right)\tan\phi - \frac{\pi}{2N}} \quad (10)$$

$$U = \frac{\pi}{4} + (\alpha - \gamma)\tan\phi + \frac{\gamma}{\cos\phi} \quad (11)$$

$$V = \gamma - \alpha \quad (12)$$

$$X(\theta) = M_n(P\cos\theta + Q\sin\theta) \quad (13)$$

$$Y(\theta) = M_n(-P\sin\theta + Q\cos\theta) \quad (14)$$

$$\theta_{min} \leq \theta \leq \theta_{max} \quad (15)$$

$$\theta_{min} = \frac{2}{N}[U + (V + X)\cot\phi] \quad (16)$$

$$\theta_{max} = \frac{2U}{N} \quad (17)$$

$$P = \frac{\gamma}{1-\left(u-\frac{n\theta}{2}\right)} \quad (18)$$

$$Q = \frac{2\gamma}{i} \times \left(\frac{v+x}{2u-n\theta}\right) + V + \frac{N}{2} + X \quad (19)$$

$$m_B = \frac{t_R}{h_t} \quad (20)$$

$$i = \frac{z_2}{z_1} \quad (21)$$

$$d_1 = M_n z_1 d_2 = M_n z_2 \quad (22)$$

$$d_{a1} = d_1 + 2M_n + 2x_1 M_n \quad (23)$$

$$d_{a2} = d_2 + 2M_n + 2x_2 M_n \quad (24)$$

$$r_{a1} = \frac{d_{a1}}{2} \quad (25)$$

$$r_{a2} = \frac{d_{a2}}{2} \quad (26)$$

$$d_{b1} = d_1 \cos\emptyset_1 \quad (27)$$

$$d_{b2} = d_2 \cos\emptyset_1 \quad (20)$$

$$r_{b1} = \frac{d_{b1}}{2} \quad (29)$$

$$r_{b2} = \frac{d_{b2}}{2} \quad (30)$$

$$C_e = \left(\frac{d_1 + d_2}{2}\right) + (x_1 + x_2)M_n \quad (31)$$

$$\propto' = \cos^{-1}\left(\frac{(d_1 + d_2)\cos\propto_n}{2C_e}\right) \quad (32)$$

$$r_{HPSTC} = \left(\left(\left((r_{a1} + r_{a2})\tan\alpha' - \sqrt{(r_{a1}^2 - r_{b1}^2)}\right) + \pi M_n \cos\alpha_n\right)^2 + r_{b1}^2\right)^{\frac{1}{2}} \quad (33)$$

$$\propto_L = \vartheta - \gamma \quad (34)$$

$$\cos\vartheta = \frac{r_b}{r_{HPSTC}} \quad (35)$$

$$\tan\gamma = \frac{y}{x} \quad (36)$$

$$h_f = y_1 - y_2 - y_3 \quad (37)$$

$$y_3 = x\tan\propto_L \quad (38)$$

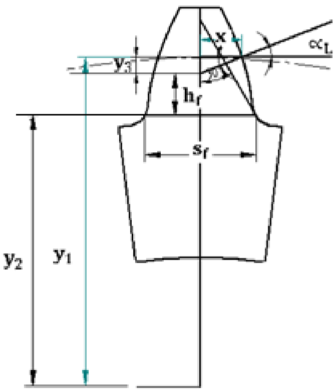

Fig. 4: Different gear tooth parameters used to estimate critical section

Using above equations and by suitable input gear data, the values of RHPSTC, load angle, critical section thickness and distance from critical section to intersection of the tooth centerline and the line of action for load at HPSTC can be obtained as shown in fig.4.

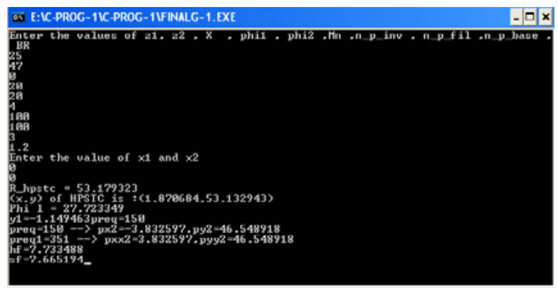

Fig. 5: Programme I/O data for gear tooth profile generation.

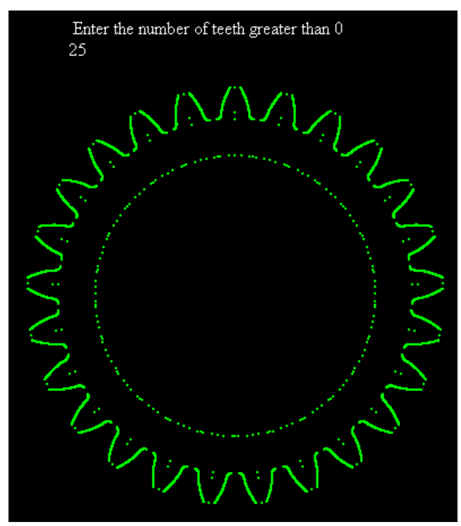

Fig. 6: Gear profile output from C-programme

Table-2: Comparison of critical section thickness for different pressure angle on drive side.

Drive side Pressure Angle	Critical Section Thickness,C-rogramme	Critical Section thickness[Ref-4]
20	3.9126	3.8572
21	3.9456	3.8949
22	3.9802	3.9333
23	4.0152	3.9726
24	4.051	4.0128
25	4.0926	4.0539
26	4.1248	4.096
27	4.1509	4.1391
28	4.2029	4.1834
29	4.2445	4.2289
30	4.3154	4.2756
31	4.3258	4.3236
32	4.3695	4.3731
33	4.4137	4.4242

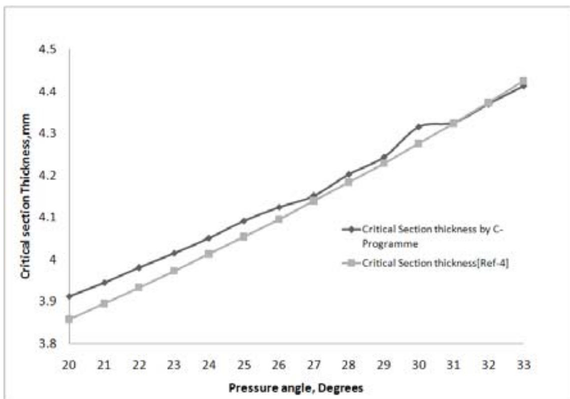

Fig. 7: Comparison of critical section.

Table-2 and fig.6: shows the comparison of critical section thickness developed by C-programme and Ref [4]. The results obtained are matches with the published one hence, for further studies in this paper C- programme o/p will be used.

4 Finite element analysis procedure

As a major part of present investigation a series of finite element analyses has been carried out for different sets of symmetric and asymmetric spur gears listed in table.1, subjected to a load at highest point of single tooth of contact (HPSTC).Gears are used to transmit a power of 18KW at 1600 rpm. Key points for involute spur gears were generated using "C "programme and same can be used for generating model for ANSYS as shown in fig.7.

A finite element problem is treated as plane stress with thickness problem and a plane 182, 8-noded quadrilateral element are used to discritized the gear tooth domain.

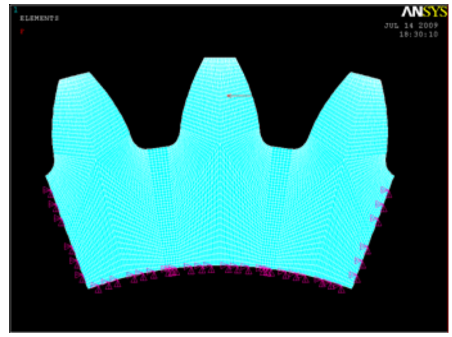

Fig. 8: Gear tooth system considered for finite element analysis with proper boundary conditions

The first investigation involved a two–dimensional plane stress analysis for 4 mm module and 20^0 pressure angle on both sides of the gears with 25 teeth and zero profile shifts. The gear tooth is considered to be a cantilever and it is constrained at the rim, an element supports the two degree of freedom and all the degrees of freedom are fixed. The gear tooth is loaded at HPSTC. The above meshed model, which is subjected to the boundary conditions and loading were statically analyzed and software performs the mathematical calculations and results are obtained in the post processing stage.

Similar analyses were carried out for different pressure angles on drive side. In the post- processor stage accepts the results and generates the contour plots for bending stress at the critical section, Vonmoises stress and displacements.

5 Bending stress calculation

The stress that takes a very important part in tooth root crack organization is considered to be maximum tensile stress σ_t at the root section which is to be determined from the following equation for both symmetric and asymmetric spur gear tooth. Stress results obtained from these equations are very nearer to the finite element results obtained from the ANSYS software.

$$\sigma_{t\,max} = \frac{F_t}{b\,m} K_z\, K_\lambda\, K_\alpha \tag{39}$$

$$K_z = 3.56 + 4.59\left(\frac{1}{z}\right) + 1.740\left(\frac{1}{z}\right)^3 \tag{40}$$

$$K_\lambda = \exp\left[\left\{2.5\left(\frac{1}{z}\right) - 0.5\right\}\frac{\lambda}{m}\right] \tag{41}$$

$$K_\alpha = 1.32 - 1.82\times10^{-2}\alpha + 1.17\times10^{-4}\alpha^2 \tag{42}$$

6 Results and discussions

The results mainly consist of bending stress at critical section and displacements at the centre line of the gear tooth for different pressure angles.

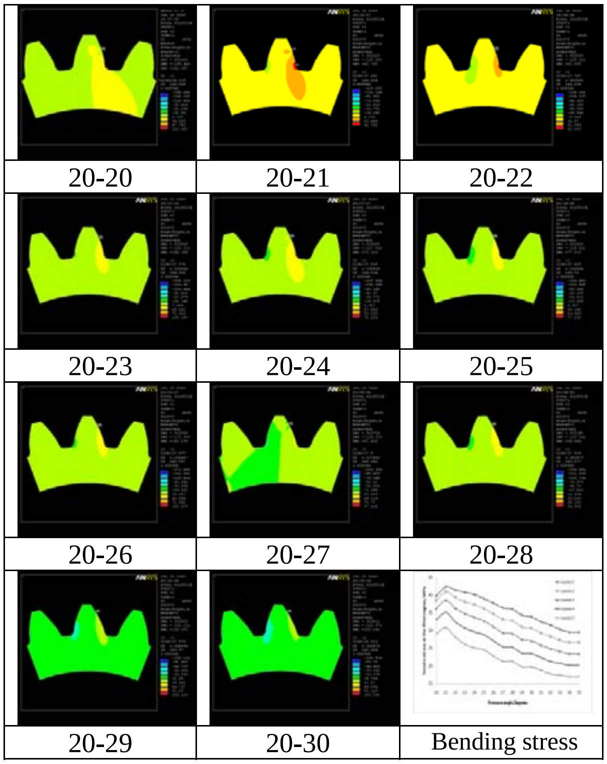

Fig. 9: Bending stress contour for different pressure

Above figure shows that the distribution of bending stress for different pressure angles on drive side, distribution pattern is same for all pressure angles.

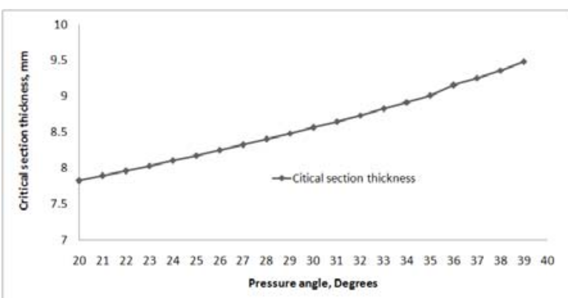

Fig. 10: Critical section thickness for different pressure

The fact that as the pressure angle increases, with increase in the tooth thickness at the critical section, and hence fillet region becomes stronger on drive side.

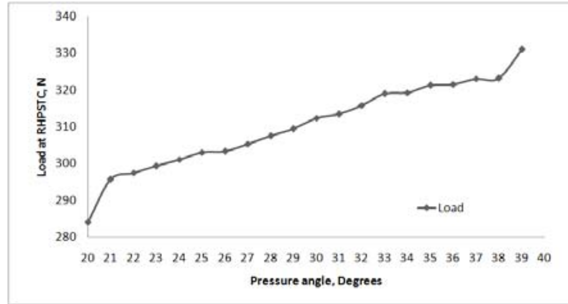

Fig. 11: Load at RHPSTC for different pressure angles.

As the pressure angle increases radius at highest point for single tooth of contact increase, which increase the moment on the gear drive.

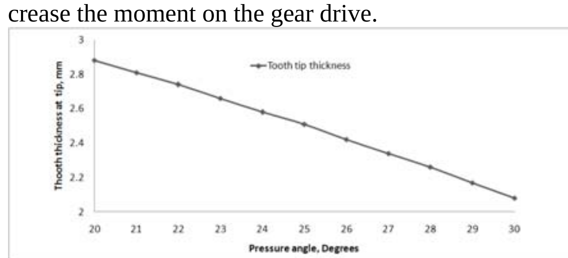

Fig. 12: Tooth thickness at the tip for different pressure angles.

It is evident from the figure that as the pressure angle increases tooth thickness at the tip reduces.

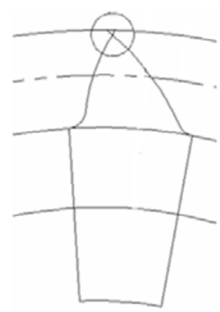

Fig. 13: Gear tooth profile for $20^0/45^0$ pressure angle

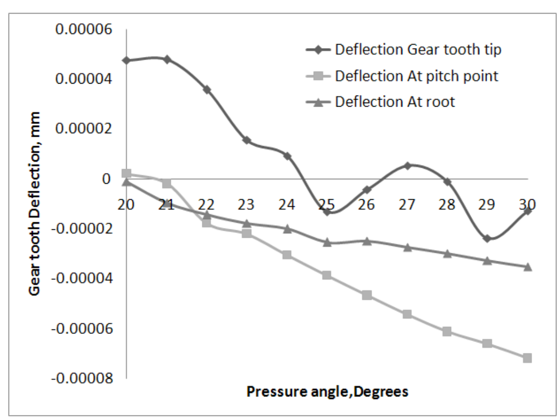

Fig. 14: Gear tooth deflection at the gear centre line

Further increase in the pressure angle on the drive side gear tooth becomes pointed and deflection at the gear tooth tip increases (Fig. 13) and there is a chance of tooth breakage, and stress at the critical section shifts from gear tooth root to the tip. Hence pressure angle on the drive side is preferred up to 35^0.

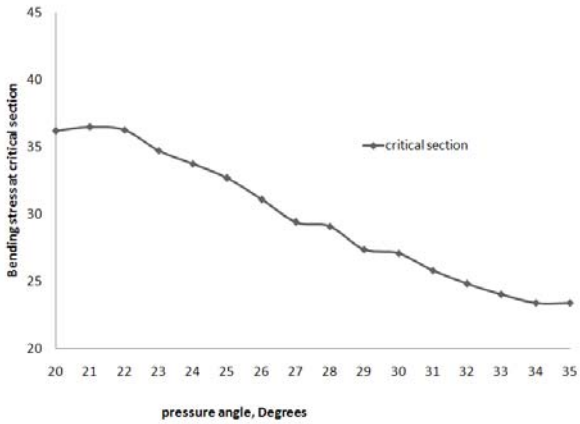

Fig. 15: Bending stress at the critical section for different pressure angles on drive side.

It was found that the bending stress at the critical section reduces drastically with increases in the pressure angle on the drive side for certain limit.

7 Conclusions

From the above results following conclusions can be drawn:

- The limiting number of teeth to avoid the undercutting is lowered. That is to say if the pressure angle is increased, pinions with comparatively lesser number of teeth can be generated without undercutting
- The shape of the tooth becomes more pointed or peaked
- Tooth flank becomes more curved
- The relative sliding velocity is reduced
- Wear is less
- Tooth load carrying capacity increases

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