Effect of Rim Thickness on Symmetric and Asymmetric Spur Gear Tooth Bending Stress

G. Mallesh[1*], Dr. V B Math[2], Ravitej[3], Krishna Prasad Bhat P[3], Paramesh Kumar M K[3]

[1] Assistant Professor, Sri Jayachamarajendra College of Engineering, Mysore-570006, India
[2] Professors, Basaveshwar Engineering College, Bagalkot-587101, India
[3]Research Group, Sri Jayachamarajendra College of Engineering, Mysore-570006, India

G.Mallesh (Email: mallesh_gowder@yahoo.co.in)

Abstract

Thin rim gears find application in high-power, lightweight aircraft transmissions. Bending stresses in thin rim spur gear tooth fillets and root areas differ from the stresses in solid gears due to rim deformations. Rim thickness is a significant design parameter for these gears. The rim thickness factor is used in the situations in which a gear is made with a rim and spokes rather than a solid disc. Under these circumstances, failure can occur across the rim rather than through the tooth root.

An asymmetric spur gear drive means that larger and smaller pressure angles are applied for the driving and coast sides. The two profiles of a gear tooth are functionally different for most gear drives. The workload on one side of profile is significantly higher than the other Gears.

The main objective of this paper is to estimate the critical section for different pressure angles and backup ratios using computer programme and compare the results obtained by other researchers. Developed programme is used to create a finite element model for symmetric and asymmetric spur gear tooth to study the effect of bending stress at the critical section for different backup ratios. To study the effect of above parameter ANSYS was used .The rim thickness was varied and the location and magnitude of the maximum bending stresses were reported and results obtained were compared with the Lewis bending equation.

Keywords: Backup Ratio, Asymmetric Spur Gear Tooth, Finite Element Analysis, Bending Stress

1 Introduction

Gears are the most common means of transmitting power in the modern mechanical engineering world. A gear can be defined as a machine element used to transmit motion and power between rotating shafts by means of progressive engagement of projections called teeth. Gears have a wide variety of applications which vary from a tiny size used in watches to the large gears used in lifting mechanisms and speed reducers. They form vital elements of main and ancillary mechanisms in many machines such as automobiles, tractors, metal cutting machine tools etc.

In recent times, the gear design has become a highly complicated and comprehensive subject. A designer of a modern gear drive system must remember that the main objective of a gear drive is to transmit higher power with comparatively smaller overall dimensions of the driving system which can be constructed with the minimum possible manufacturing cost, runs reasonably free of noise and vibration, and which required little maintenance. He has to satisfy, among others the above conditions and design accordingly, so that the design is sound as well as economically viable.

Present day gears are subjected to the different types of failures like fracture under bending stress, surface failure under internal stress etc.One major cause of gear failure is fracture at the base of the gear tooth due to bending fatigue. Design models for this mode of failure use a parabolic beam with stress concentration correction. The bending strength is influenced by: the gear size, described by the diametral pitch; the shape of the tooth, described by the number of teeth on the gear; the highest location of the full load, described by the number of teeth on the mating gear; and the fillet geometry of the gear tooth. For thin rim gears, the thickness of the rim is another significant factor which influences the bending strength of the gear. Rim deflections increase the bending stresses in the tooth fillet and root areas. Therefore in aircraft applications, the rim thickness and allowable stress are optimized to achieve light weight.

Wilcox and Coleman applied the finite element method to analyze the bending stresses in a gear tooth of a solid gear and demonstrated good agreement with photoelastic stress measurements. For thin rim gearing, Drago et al. studied rimmed gear stresses experimentally with strain gages and photoelastic models and analytically with two and three-dimensional finite element models. Their studies report a nearly constant bending stress as the rim thickness decreases and a sudden increase in bending stress below a certain rim thickness. Analytical studies have been conducted on thin rim gear stresses with finite elements by several researchers. Oda et al.

studied a single tooth model of a thin rim spur gear using a five tooth segment fixed at its sides. They used strain gages to verify their results. Arai et al. studied a spoked thin rim gear with four teeth in the free rim arc between spokes. Chang et al. applied a two-dimensional finite element grid to a single thin rim tooth with fixed constraints at the tooth sides to demonstrate the stress distribution in the tooth. Chong et al. used two dimensional triangular finite elements and a rack model to study the effects of the rim on the bending stress in the fillet. Their rack model had statically determinate beam supports on segments of different lengths. Von Eiff et al. used a finite element model of a three tooth segment for both external and internal gears to study the maximum bending stresses at the root of the central tooth. Gulliot and Tordion analyzed the problem of a thin rim on a support hub using the finite element method. All of these studies report a nearly constant tensile bending stress as the rim thickness decreases to a value near the tooth depth. The tensile root stress increases rapidly with further reductions of rim thickness. However, each study reported a different transition rim thickness value. These studies also differed in the rim support geometry and the number of teeth on the gear. The ring flexibility of the rim influences both the tooth stiffness 11and the location and magnitude of the maximum bending stress in a thin rim gear. Thus, the support constraints affect the maximum bending stress.

In this paper a three teeth segment of a 25 tooth gear in mesh with a 47 tooth asymmetric spur gear for different pressure angles on drive and coast sides with different backup ratios were studied. To study the effect of above parameter ANSYS was used .The rim thickness was varied and the location and magnitude of the maximum bending stresses were reported.

2 Asymmetric spur gear teeth

The two profiles (sides) of a gear tooth are functionally different for many gears. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of the asymmetric tooth shape reflects this functional difference.

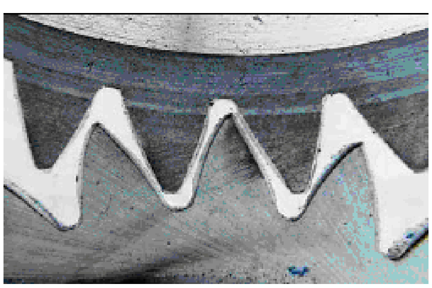


Fig. 1: Asymmetric spur gear.

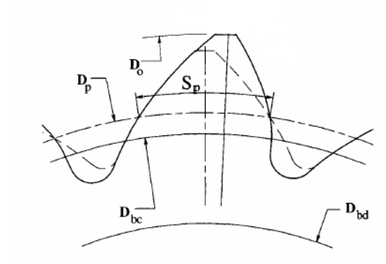


Fig. 2: Asymmetric spur gear with different base circles.

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile. The opposite profile is typically unloaded or lightly loaded during relatively short work periods. The degree of asymmetry and drive profile selection for these gears depends on the application.

The difference between symmetric and asymmetric tooth is defined by two involutes of two different base circles D_{bd} and D_{bc}. The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter D_p that should be bigger than the largest base diameter.

Asymmetric gears simultaneously allow an increase in the transverse contact ratio and operating pressure angle beyond the conventional gear limits. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles by changing the coast side profiles. This provides higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

3 Backup ratio

The rim thickness is not sufficient to provide full support for the tooth root, the location of bending fatigue may be through the gear rim rather than at the root fillet or critical section. In such cases, the use of a stress modifying factor K_B is recommended. This factor, the rim thickness factor $K_{B,}$ adjust the bending stress for thin rimmed gear. It is the function of backup ratio.

It is defined as the ratio of rim thickness below the tooth root to the whole depth of the tooth

$$m_B = \frac{t_R}{h_t}$$

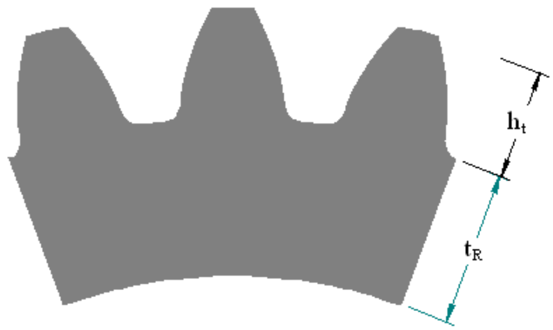


Fig. 3: Spur gear tooth with rim and tooth thickness

4 Involute gear tooth profile generation.

With the emergence of computers, engineering modeling and analysis is getting more dependent on computers day by day. Computerized process involves many production systems and engineering procedures. In a product design process involving engineering analysis, design alternative has been developed in the geometric modeling process. Different parameters listed in Table 1.are used to generate gear tooth profile using C-Programming.

Table-1: Gear tooth parameters

Sl.No.	Description	Value
1	Back up ratio	0.6, 0.7, 0.8, 0.9,1.0, 1.2,1.4,1.6 & solid gear
2	Number of teeth	25 and 47
3	Module	4 mm
4	Pressure angle, Coast side	20^0 fixed
5	Pressure angle, Drive side	20^0-30^0 increment by 1^0
6	Profile shift factor	0.0

Equation used to generate spur gear tooth profile with backup ratio.

$$\bar{r}(\theta) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \quad (2)$$

$$X(\theta) = N\frac{M_n}{2}\left\{\sin\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right] \cos(\phi + \theta)\right\} \quad (3)$$

$$Y(\theta) = N\frac{M_n}{2}\left\{\cos\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right] \sin(\phi + \theta)\right\} \quad (4)$$

$$\theta_{min} \leq \theta \leq \theta_{max} \quad (5)$$

$$\theta_{min} - \frac{2}{N}\left[U + (V + X)\cot\phi\right] \quad (6)$$

$$\theta_{max} = \frac{1}{N\cos\phi} \times \sqrt{(2 + N + 2X)^2 - N(\cos\phi)^2} - \left(1 + \frac{2X}{n}\right)\tan\phi - \frac{\pi}{2N} \quad (7)$$

$$U = \frac{\pi}{4} + (\alpha - \gamma)\tan\phi + \frac{\gamma}{\cos\phi} \quad (8)$$

$$V = \gamma - \alpha \quad (9)$$

$$\bar{r}(\theta) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \quad (10)$$

$$X(\theta) = M_n(P\cos\theta + Q\sin\theta) \quad (11)$$

$$Y(\theta) = M_n(-P\sin\theta + Q\cos\theta) \quad (12)$$

$$\theta_{min} \leq \theta \leq \theta_{max} \quad (13)$$

$$\theta_{min} = \frac{2}{N}\left[U + (V + X)\cot\phi\right] \quad (14)$$

$$\theta_{max} = \frac{2U}{\gamma} \quad (15)$$

$$P = \frac{\gamma}{1+\left(u-\frac{n\theta}{2}\right)} \quad (16)$$

$$Q = \frac{2\gamma}{L} \times \left(\frac{v+x}{2u-n\theta}\right) + V + \frac{N}{2} + X \quad (17)$$

$$m_B = \frac{t_r}{h_t} \quad (18)$$

Above equation are used to generate involute profile and fillet radius as shown in fig.7. Profiles generated using above equations are good agreement with the earlier publications.

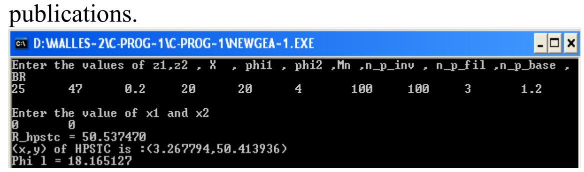


Fig. 4: Programme input data for profile generation

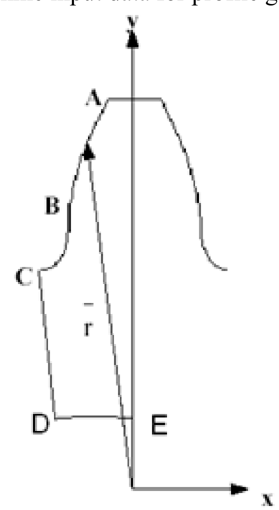


Fig. 5: Generated involute profile and fillet radius

5 Finite element analysis procedure

As a major part of present investigation a series of finite element analyses has been carried out for different sets of symmetric and asymmetric spur gears listed in table.1, subjected to a load at highest point of single tooth of contact (HPSTC).Gears are used to transmit a power of 18KW at 1600 rpm. Key points for involute spur gears were generated using “C “programme and same can be used for generating model for ANSYS as shown in fig.8.

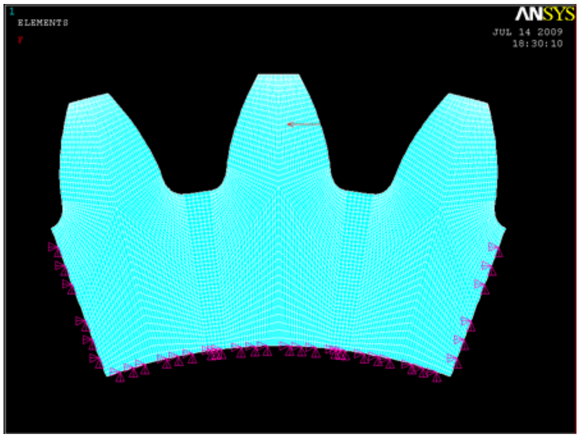


Fig. 6: Gear tooth system considered for finite element analysis with proper boundary conditions

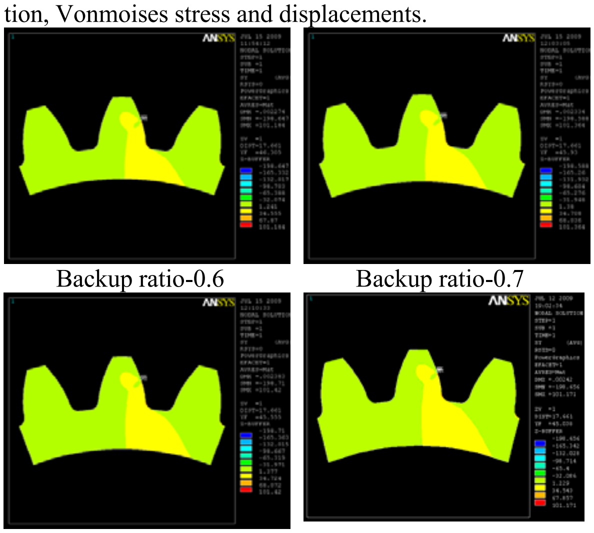
A finite element problem is treated as plane stress with thickness problem and a plane 182, 8-noded quadrilateral element are used to discritized the gear tooth domain.

The first investigation involved a two–dimensional plane stress analysis for 4 mm module and 20^0 pressure angles on both sides of the gears with 25 teeth on pinion with backup ratio 1.2. The gear tooth is considered to be a cantilever and it is constrained at the rim, an element supports the two degree of freedom and all the degrees of freedom are fixed. The gear tooth is loaded at HPSTC. The above meshed model, which is subjected to the boundary conditions and loading were statically analyzed and software performs the mathematical calculations and results are obtained in the post processing stage.

Similar analyses were carried out for different pressure angles on drive side and backup ratios. In the post- processor stage accepts the results and generates the contour plots for bending stress at the critical section, Vonmoises stress and displacements.

6 Results and discussions

A result mainly consists of bending stress at critical section, Vonmoises stress and displacements.



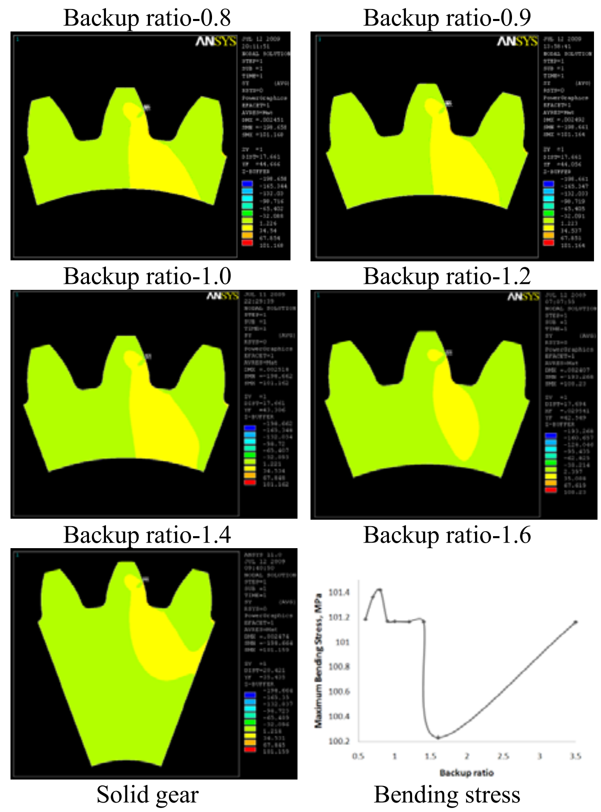


Fig. 7: Maximum bending stress for different backup ratios.

From fig.7: a constant bending stress was developed as the rim thickness decreased to value nearer to the tooth depth, the tensile tooth stress increases rapidly with the further increase or decrease in the backup ratio.

It is concluded in [2] that gears with backup ratios of 3.5 and 1.0 produced tooth fractures while the backup ratios of 0.3 and 0.5 produced rim fractures. Hence we recommend that backup ratio of 1.6 is optimum from the results obtained by the analysis.

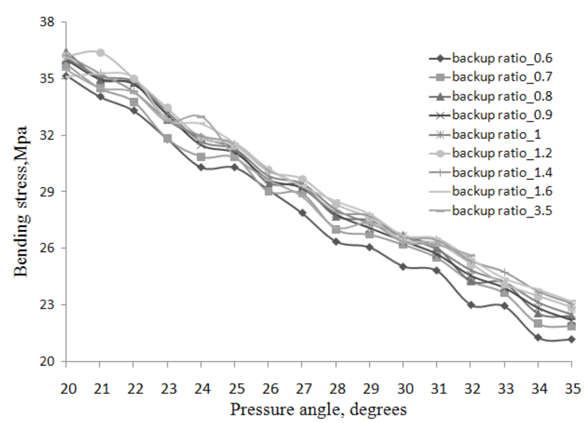


Fig. 8: Bending stress at the critical section for different pressures angles.

Fig. 8: shows the distribution of bending stress in the critical section for different pressure angles and backup ratios. It indicates that as pressure angle increases bending stress at the critical section decreases.

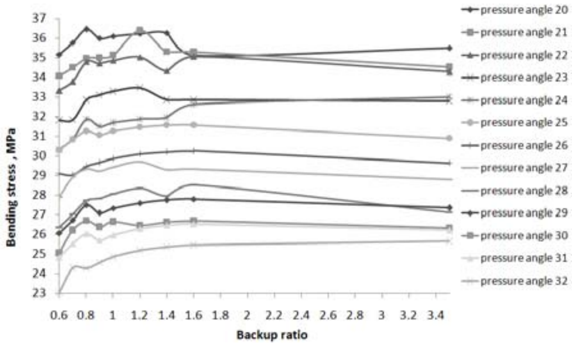


Fig. 9: Bending stress at the critical section for different backup ratios.

Bending stress at the critical section increases with decrease in the value of backup ratio from 0.5 to 0.3[2] and increases with increase in the backup ratio from 0.6 to 1.6 and remains constant thereafter.

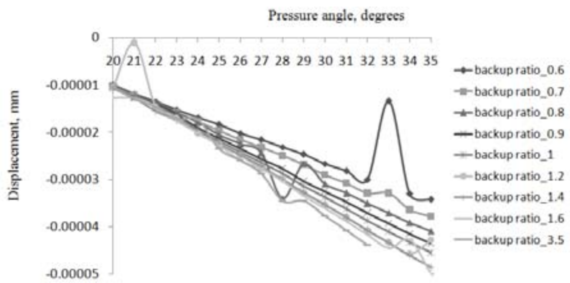


Fig. 9: Displacement at the asymmetric spur gear tooth root for different backup ratios.

For backup ratios 0.6 and 0.7, deflection increases at the gear tooth root which indicates that there is a chance of rim fracture.

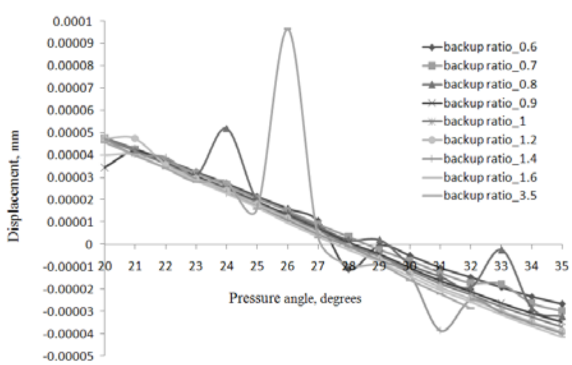


Fig. 10: Displacement at the asymmetric spur gear tooth tip for different backup ratios.

As the backup ratio increases above 1.6, tooth deflection will be less with the increase in the pressure angle.

5 Conclusions

From the above results following conclusions were drawn:

- The bending stress at the critical section is maximum for backup ratios above 3.5 and minimum for 1.6 in asymmetric spur gear tooth. AGMA recommends 1.2 backup ratios for symmetric spur gear tooth.
- For backup ratios 0.6 and 0.7, deflection increases at the gear tooth root which indicates that there is a chance of rim fracture.
- Bending stress are primarily depends on rim support geometry. The stiffer the rim support, the lower is the backup ratio at which the stresses increase over those of a similar solid gear.
- These types of gears find application in high-power, light-weight aircraft transmissions.

Acknowledgment

We thank the management, Principals and Heads of the Department of Mechanical Engineering, Sri Jayachamarajendra College of Engineering, Mysore and Basaveshwar Engineering College, Bagalkot, for providing us an opportunity and encouraging to present this research paper.

References

[1]. G.D Bibe, S.K.Reddy, M. Savage, R.E Handschuh, "Effects of Rim Thickness on Spur Gear Bending Stress", *NASA Technical Memorandum 104388, 27th Joint Propulsion Conference Cosponsored by the AIAA, SAE, and ASME Sacramento, California,* June 24-26, 1991.

[2]. David G. Lewicki, Roberto Ballarini, "Effect of Rim Thickness on Gear Crack Propagation Path", *NASA, Technical Memorandum 107229, Seventh International Power Transmission and Gearing Conference, Sponsored by the American Society of Mechanical Engineers,* October 6-9, 1996 San Diego, California.

[3]. Lewicki D.G., Ballarini R.,"Gear Crack Propagation Investigations",*International Journal of Fatigue,* Vol. 19, Issue: 10, 1997. PP. 731

[4]. Lewicki David G., Ballarini Roberto, "Rim Thickness Effects on Gear Crack Propagation Life", *International Journal of Fracture,* Vol. 87, Issue: 1, September 1997. PP. 59 – 86.

[5]. Shuting Li, "Deformation and Bending Stress Analysis of a Three-dimensional Thin-rimmed Gear", *Journal of Mechanical Design***,** March 2002, Vol. 124' 129.

[6]. Alexander L. Kapelevich and Yuriy V. Shekhtman,"Direct Gear Design: Bending Stress Minimization" *Gear Technology,* September/October 2003, PP, 45-47

[7]. Kramberger J., Sraml M., Potrc I., Flasker J., — "Numerical Calculation of Bending Fatigue Life of

Thin-Rim Spur Gears*", Engineering Fracture Mechanics,* Vol. 71, Issue: 4-6, March - April, 2004. PP. 647-656.

[8].Yang, Shyue-Cheng "Mathematical Model of a Helical Gear with Asymmetric Involute Teeth and its Analysis", *The International Journal of Advanced Manufacturing Technology,* Vol. 26, Issue: 5-6, September 2005. PP. 448 – 456.

[9]. Yesh P. Singh, Ravichandra Patchigolla, "Effect of Rim Thickness on Bending Stresses in Low Addendum Large Spur Gears*", International ANSYS Conference,* 2006

[10]. M. N. Bettaieb, P. Velex, M. Ajmi**, "**A Static and Dynamic Model of Geared Transmissions by Combining Substructures and Elastic Foundations"Applications to Thin-Rimmed Gears, *Transactions of the ASME,* Vol. 129, February 2007.