Effect of Tooth Profile Modification In Asymmetric Spur Gear Tooth Bending Stress By Finite Element Analysis

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Abstract

Gearing is one of the most critical components in a mechanical power transmission system, and in most industrial rotating machinery. It is possible that gears will predominate as the most effective means of transmitting power in future machines due to their high degree of reliability and compactness. In addition, the rapid shift in the industry from heavy industries such as shipbuilding to industries such as automobile manufacture and office automation tools will necessitate a refined application of gear. Presently gears are suffered by backlash, undercut and interference. These defects can be eliminated by increasing the pressure angle and increasing addendum of mating gears. An additional alteration that is very rarely used is to make the gears asymmetric with different pressure angles for each side of the tooth. This is because two profiles of a gear tooth are functionally different for most of the gear drives. The workload on one side of profile is significantly higher than the other side of the gear. An asymmetric spur gear drive means that larger and smaller pressure angles are applied for the driving and coast sides.

Interference is a serious defect in the involute system of gearing and should be avoided by undercutting the tooth, when the number of teeth is less than the minimum required number of teeth. Apart from the fact that interference hampers the conjugate action when the involute portion of a tooth mates with the non involute portion of the mating tooth, the two meshing gears will not have free rotation. Rather, the gear causing interference will have a tendency to jam on the flank of the pinion unless, of course, the pinion tooth-root has already been undercut making room to provide free movement of the gear tooth. Besides, due to interference and in the absence of undercut, the mating gear will try to scoop out metal from the interfering portion. Therefore, the teeth become damaged and it will have an overall detrimental effect on the gearing system.

The main objective of this paper is to study the effect of bending stress at the critical section for different pressure angles on the drive side along with the profile shift. Comparison has been made for symmetric and asymmetric spur gear tooth using Lewis equation and Finite element analysis software.

Keywords: Asymmetric spur gear, Profile shift, Finite element method

1 Introduction

In engineering and technology the term "gear" is defined as a machine element used to transmit motion and power between rotating shafts by means of progressive engagement of projection called teeth.

Invention of the gear cannot be attributed to one individual as the development of the toothed gearing system evolved gradually from the primitive form when wooden pins were arranged on the periphery of simple, solid, wooden wheels to drive the opposite member of the pair. These wheels served the purpose of gears in those days. Although the operation was neither smooth nor quiet, these were not important consideration as the speeds were very low. The motive power to turn these systems was generally provided by treadmills, which were operated by men, animals, water wheels or windmills.

In recent times, the gear design has become a highly complicated and comprehensive subject. A designer of a modern gear drive system must remember that the main objective of a gear drive is to transmit higher power with comparatively smaller overall dimensions of the driving system which can be constructed with the minimum possible manufacturing cost, runs reasonably free of noise and vibration, and which required little maintenance. He has to satisfy, among others the above conditions and design accordingly, so that the design is sound as well as economically viable.

Present day gears are subjected to the different types of failures like fracture under bending stress, surface failure under internal stress etc. These failures are mainly due to backlash, undercutting and interference.

Backlash: The amount by which the width of a tooth space exceeds the thickness of the engaging tooth on the pitch circles. (Fig. 1a)

Undercut: A condition in generated gear teeth when any part of the fillet curve lies inside of a line drawn tangent to the working profile at its lowest point. (Fig. 1b)

Interference: Important aspect of kinematics of gearing is interference. When the gear tooth tries to dig below the base circle of mating gear then the gear tooth action shall be non conjugate and violate the fundamental law of gearing this non conjugate action is called the interference. (Fig. 1c)

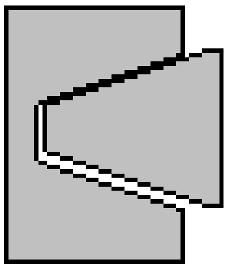


Fig. 1:a Backlash

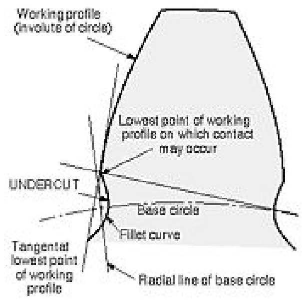


Fig. 1:b Undercut

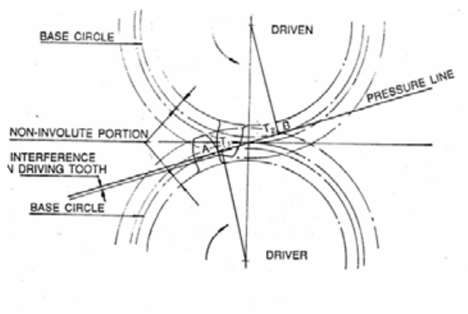


Fig. 1:c Interference

Fig. 1: Different types of defects in spur gears.

These defects can be eliminated by:

- Under cutting can be avoided by increasing the pressure angle.
- Backlash and interference can be avoided by increasing the addendum of mating gear
- Another way of increasing the load capacity of transmissions is to modify the involute geometry. This has been a standard practice in sophisticated gear design for many years. The nomenclature describing these types of gear modifications can be quite confusing with reference to addendum modification or profile shift.
- An additional alteration that is very rarely used is to make the gears **asymmetric** with different pressure angles for each side of the tooth.

2 Asymmetric spur gear teeth

The two profiles (sides) of a gear tooth are functionally different for many gears. The workload on one profile is significantly higher and is applied for longer periods of time than for the opposite one. The design of the asymmetric tooth shape reflects this functional difference.

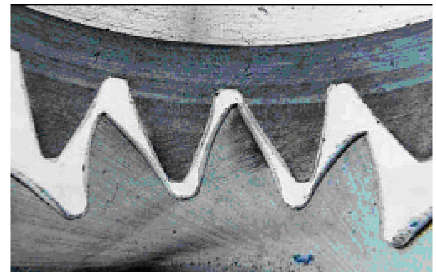


Fig. 2: Asymmetric spur gear.

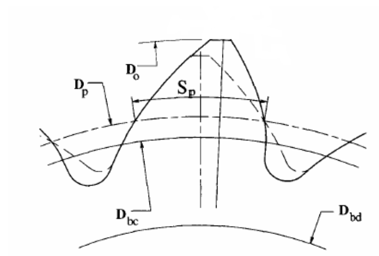


Fig. 3: Asymmetric spur gear with different base circles.

The design intent of asymmetric gear teeth is to improve the performance of the primary contacting profile. The opposite profile is typically unloaded or lightly loaded during relatively short work periods. The degree of asymmetry and drive profile selection for these gears depends on the application.

The difference between symmetric and asymmetric tooth is defined by two involutes of two different base circles D_{bd} and D_{bc}. The common base tooth thickness does not exist in the asymmetric tooth. The circular distance (tooth thickness) S_p between involute profiles is defined at some reference circle diameter D_p that should be bigger than the largest base diameter.

Asymmetric gears simultaneously allow an increase in the transverse contact ratio and operating pressure angle beyond the conventional gear limits. Asymmetric gear profiles also make it possible to manage tooth stiffness and load sharing while keeping a desirable pressure angle and contact ratio on the drive profiles by changing the coast side profiles. This provides higher load capacity and lower noise and vibration levels compared with conventional symmetric gears.

3 Profile shift

The height of the tooth above the pitch circle or the radial distance between the tip diameter and the pitch diameter is called addendum. When gears are produced by a generating process, the datum line of the basic rack profile need not necessarily form a tangent to the reference circle; the tooth form can be altered by shifting the datum line from the tangential position. The involute shape of the tooth profile is retained. The radial displacement from the tangential position is termed addendum modification factor or profile shift.

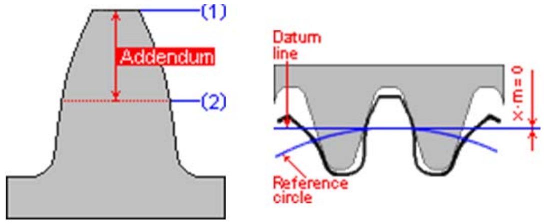
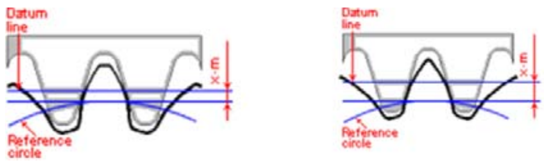


Fig. 4: Profile shift in gears

There are two different types of profile shifts based on the movement of the cutter from the reference line as shown in fig.5



Positive profile Negative profile

Fig. 5: different types of profile shift

The displacement is considered positive in the direction away from the centre of the gear. The displacement is considered negative in the direction towards the centre of the gear. The load carrying capacity of the teeth can be improved only by the positive profile shift.

Theoretical expression of the correction factor is

$$x = \frac{Z - Z_{\min}}{Z_{\min}}$$

(1)

$Z_{\min}$ is a minimum number of teeth to avoid undercutting. For $\alpha=20^0$, we know that the theoretical value of $Z_{\min}$ is 17. It has also been pointed out that a slight undercutting does not affect tooth action.

Hence, we consider the positive profile shift for analyzing the bending stress at the critical section.

4 Involute gear tooth profile generation.

With the emergence of computers, engineering modeling and analysis is getting more dependent on computers day by day. Computerized process involves many production systems and engineering procedures. In a product design process involving engineering analysis, design alternative has been developed in the geometric modeling process. Different parameters listed in Table 1.are used to generate gear tooth profile using C- Programming.

Table-1: Gear tooth parameters

Sl.No.	Description	Value
1	Profile shift factor	0,0.1,0.2,0.3,0.4,0.5
2	Number of teeth	25 and 47
3	Module	4 mm
4	Pressure angle, Coast side	20^0 fixed
5	Pressure angle, Drive side	20^0-30^0 increment by 1^0
6	Back up ratio	1.2

Equation used to generate spur gear tooth profile

$$\bar{r}(0) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \tag{2}$$

$$X(\theta) = N\frac{M_n}{2}\left\{\sin\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\cos(\phi+\theta)\right\}$$

$$Y(\theta) = N\frac{M_n}{2}\left\{\cos\theta - \left[\left(\theta + \frac{\pi}{2N}\right)\cos\phi + \left(\frac{2X}{N}\right)\sin\phi\right]\sin(\phi+\theta)\right\} \tag{4}$$

$$\theta_{min} \le \theta \le \theta_{max} \tag{5}$$

$$\theta_{min} = \frac{2}{N}[U + (V + X)\cot\phi] \tag{6}$$

$$\theta_{max} = \frac{1}{N\cos\phi} \times \sqrt{(2 + N + 2X)^2 - N(\cos\phi)^2} - \left(1 + \frac{2X}{n}\right)\tan\phi - \frac{\pi}{2N} \tag{7}$$

$$U = \frac{\pi}{4} + (\alpha - \gamma)\tan\phi + \frac{\gamma}{\cos\phi}$$

(8)

$$V = \gamma - \alpha \tag{9}$$

$$\bar{r}(\theta) = \begin{Bmatrix} x(\theta) \\ y(\theta) \end{Bmatrix} \tag{10}$$

$$X(\theta) = M_n(P\cos\theta + Q\sin\theta) \tag{11}$$

$$Y(\theta) = M_n(-P\sin\theta + Q\cos\theta) \tag{12}$$

$$\theta_{min} \le \theta \le \theta_{max} \tag{13}$$

$$\theta_{min} = \frac{2}{N}[U + (V + X)\cot\phi]$$

(14)

$$\theta_{max} = \frac{2U}{N}$$

(15)

$$P = \frac{\gamma}{l + \left(u - \frac{n\theta}{2}\right)}$$

(16)

$$Q = \frac{2\gamma}{L} \times \left(\frac{v + x}{2u - n\theta}\right) + V + \frac{N}{2} + X \tag{17}$$

Above equation are used to generate involute profile and fillet radius as shown in fig.7. Profiles generated using above equations was good agreement with the earlier publications.

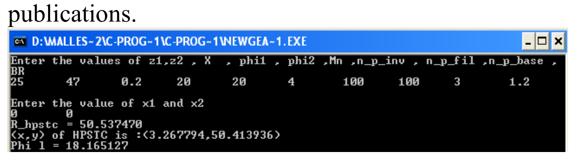


Fig. 6: Programme input data for profile generation

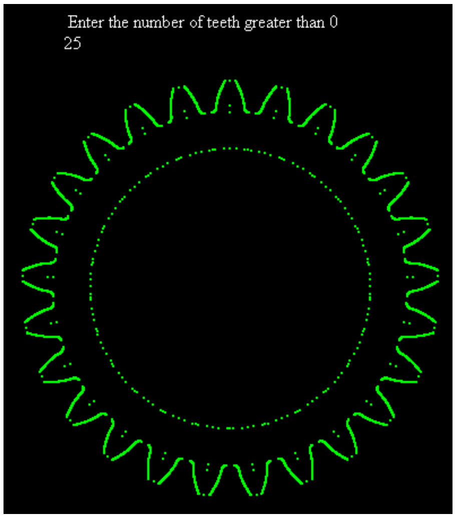


Fig. 7: Generated gear profile

5 Finite element analysis procedure

As a major part of present investigation a series of finite element analyses has been carried out for different sets of symmetric and asymmetric spur gears listed in table.1, subjected to a load at highest point of single tooth of contact (HPSTC).Gears are used to transmit a power of 15KW at 1000 rpm. Key points for involute spur gears were generated using "C "programme and same can be used for generating model for ANSYS as shown in fig.8.

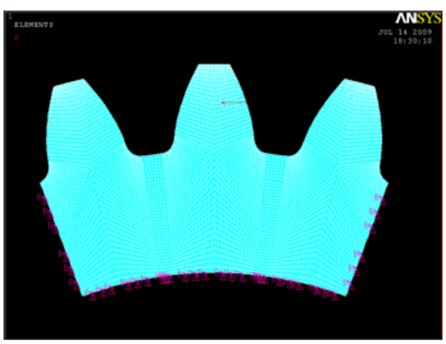


Fig. 8: Gear tooth system considered for finite element analysis with proper boundary conditions

A finite element problem is treated as plane stress with thickness problem and a plane 182, 8-noded quadrilateral element are used to discritize the gear tooth domain.

The first investigation involved a two–dimensional plane stress analysis for 4 mm module and 20^0 pressure angle on both sides of the gears with 25 teeth and zero profile shifts. The gear tooth is considered to be a cantilever and it is constrained at the rim (A-B-C-D). An element supports the two degree of freedom and all the degrees of freedom are fixed. The gear tooth is loaded at HPSTC. The above meshed model, which is subjected to the boundary conditions and loading were statically analyzed and software performs the mathematical calculations and results are obtained in the post processing stage.

Similar analyses were carried out for different pressure angles on drive side and profile shift as shown in fig.9. In the post- processor stage accepts the results and generates the contour plots for bending stress at the critical section and displacement at the tooth centreline.

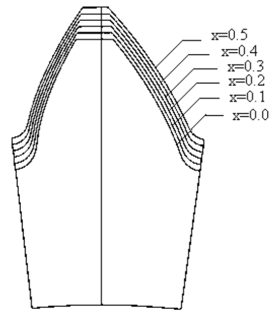


Fig.9.Asymmetric spur gear tooth for $20^0/30^0$ pressure angle with different profile shifts

6 Results and discussions

The results mainly consist of bending stress at critical section and displacements at the centre line of the gear tooth.

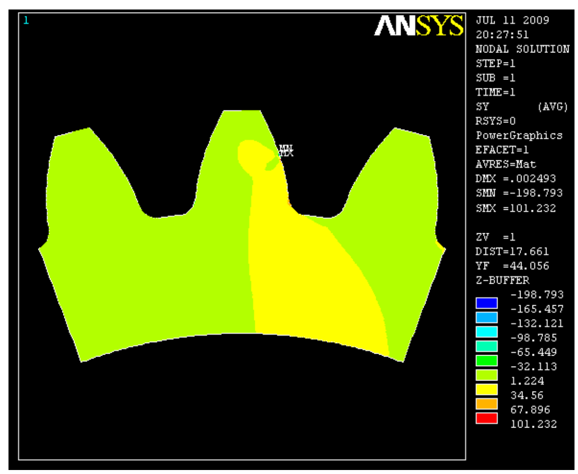


Fig. 10: Bending stress contour for x=0 profile shift.

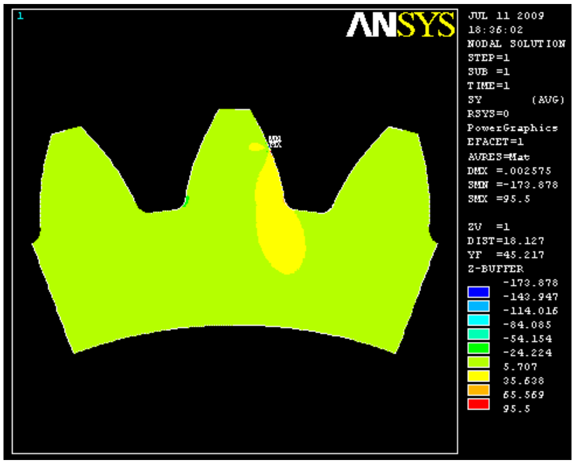


Fig. 11: Bending stress contour for x=0 .3 profile shift.

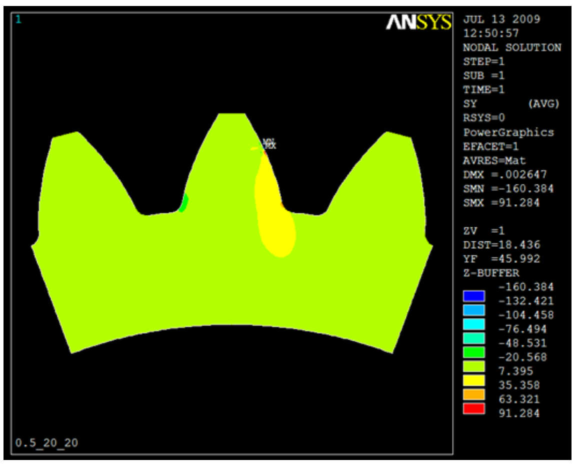


Fig. 12: Bending stress contour for x=0 .5 profile shift.

It was found that with increase in the prolife shift for a given pressure angle the bending stress of the gear tooth decreased. The above figures illustrate the same.

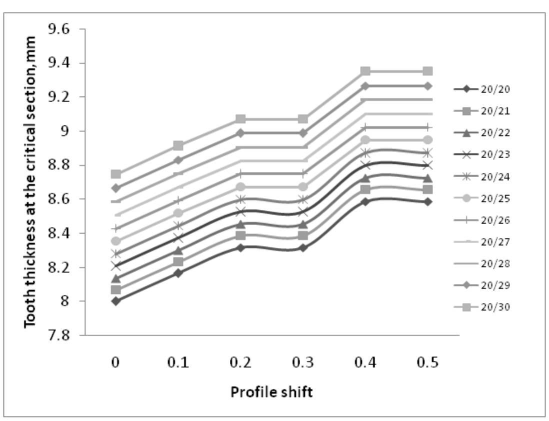


Fig. 13: Tooth thickness at the critical section for different profile shifts

With the effect of positive shift there is an increase in the tooth thickness at the critical section. It is evident from figures 9 and 13.

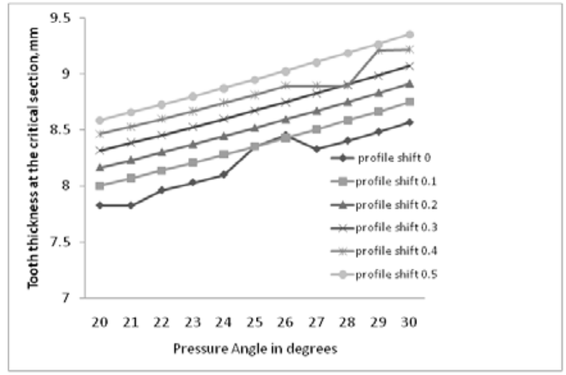


Fig. 14: Tooth thickness at the critical section for different pressure angles

Tooth thickness at the critical section also increases with an increase in the pressure angle for a given profile shift as illustrated in fig.14

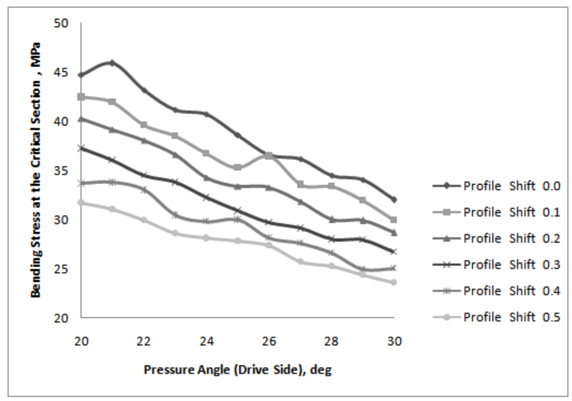


Fig. 15: Bending stress at the critical section for different pressure angles.

It was found that by increasing the pressure angle, the bending stress at the critical section decreases for a given profile shift value (fig.15). With the effect of positive shift there is a decrease in the bending stress at the critical section.

7 Conclusions

In modern usage of gear technology the correction factors are being standardized for the purpose of interchangeable gearing. Previously gears were corrected either to avoid undercutting or to achieve a predetermined centre distance. Although these reasons are still valid there are other beneficial effects which the positively corrected gear profiles offer. The advantages are

- Avoidance of undercutting.
- Attainment of predetermined centre distance.
- To increase the strength at the root and flank of the tooth. It can be shown that due to positive correction; the thickness of tooth at the root is increased,

resulting in greater load carrying capacity of the teeth. By choosing the proper amount of correction, the designer is in a position to specify gear sets of higher capacity without entailing the corresponding cost increase for materials of higher strength.

- Betterment of sliding and contact relations.
- The analysis yields that by increasing the pressure angle, the bending stress at the critical section decreases by 20-25% for a given profile shift value. With the effect of positive shift there is a reduction in the bending stress at the critical section by 20-25%.with the implementation of both profile shift and pressure angle modification, bending stress significantly decreased by 35-40%.

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