Bond Graph Modeling and Simulation of Three phase PM BLDC Motor

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Abstract

The purpose of this paper is to build a simple, accurate and fast running real time bond graph model of a three phase star-connected Permanent Magnet Brushless Direct Current (PMBLDC) Motor and implementation of the same in MATLAB environment. Brushless DC motors (BLDCM) have important advantages over brushed DC motors and induction motors. They have better speed/torque characteristics, high efficiency, and high dynamic response, are compact, need lesser maintenance.

The complete dynamic model for the BLDC motor has been systematically developed using Bond Graphs. The cause-effect relations have been analyzed and discussed. A simulation program written in MATLAB is used to verify the basic operation (performance) of the proposed topology. It is believed that the proposed model offers a reliable and low-cost solution for the emerging market of BLDC motor drives.

Key words: Brushless DC motor, bond graph modeling, MATLAB

1 Introduction

Conventional DC permanent magnet motor is the most widely used actuator, but it is associated with the problem of frequent wear out of commutator and brush. Commutation also tends to cause a great deal of electrical and RF noise. Without a commutator or brushes, a brushless motor may be used in electrically sensitive devices like audio equipments or computers. BLDC motors offer several advantages over brushed DC motors, including higher efficiency and reliability, reduced noise, longer lifetime (no brush erosion), elimination of ionizing sparks from the commutator. BLDC PM Motors are preferred actuators in automation systems owing to their many advantages. The dynamic modeling and simulation of brush less DC motor has been widely studied because of their increasing usage in industrial automation. It is therefore necessary to analyze the dynamic characteristics of a brush less dc permanent magnet motor in order to control it, simulate it, and to evaluate its performance. Many methods for obtaining the governing differential equations of dynamic system are well known. To make dynamic modeling simpler and understandable, a unified approach based upon energy transaction called Bond Graph had been invented by Henry Paynter in 1950s. The same approach is being used in this paper for modeling of brush less dc motor dynamics. The code for the system equations obtained from bond graph model of BLDC motor are written in MATLAB and then simulated by using ODE45 solver.

2 BLDC Motor Theory

As compared to a conventional DC brush motor, brushless DC (BLDC) motors are DC brush motors turned inside out, so that the field is on the rotor and the armature is on the stator. In BLDC motor, field excitation is provided by a permanent magnet and commutation is achieved electronically instead of using mechanical commutators and brushes. In BLDC motor, the mechanical ‘rotating switch’ or commutator/brush gear assembly is replaced by an external electronic switch synchronised to the rotor's position.

There are two main types of BLDC motors; trapezoidal type and sinusoidal type. In the trapezoidal motor the back-emf induced in the stator windings has a trapezoidal shape and its phases must be supplied with quasi-square currents for ripple-free torque operation. The sinusoidal motor on the other hand has a sinusoidal shaped back-emf and requires sinusoidal phase currents for ripple-free torque operation. The shape of the back-emf is determined by the
shape of the rotor magnets and the stator winding distribution.

The sinusoidal motor needs a high resolution position sensor because the rotor position must be known at every time instant for optimal operation. It also requires more complex software and hardware. The trapezoidal motor is a more attractive alternative for most applications due to its simplicity, lower price and higher efficiency [7].

3 Operation of BLDC Motor

The three phase BLDC motor is operated in a two-phases-on fashion, i.e. the two phases that produce the highest torque are energized while the third phase is off. Which two phases are energized depends on the rotor position.

Fig. 1 shows a cross section of a three-phase star-connected motor along with its phase energizing sequence. Each interval starts with the rotor and stator field lines 120° apart and ends when they are 60° apart. Maximum torque is reached when the field lines are perpendicular. The signals from the position sensors produce a three digit number that changes every 60° (electrical degrees) as shown in fig. 2 (HI, H2, H3). The figure also shows ideal current and back-emf waveforms.

![Fig. 1: BLDC motor cross section and phase energizing sequence.](image)

The most common method of sensing the rotor position in a BLDC motor is using hall-effect position sensors. For a BLDC motor with a trapezoidal back-EMF, it is sufficient to get position information that is updated at every 60 degree electrical interval, called six steps commutation. The position information is then used to decide the triggering of inverter switches. Generally, three hall-effect sensors are used for a three phase motor. Current commutation is done by a six-step inverter as shown in a simplified form in fig. 3. The switches are shown as bipolar junction transistors but MOSFET switches are more common.

![Fig. 2: Ideal back-emfs, phase currents, and position sensor signals](image)

4 Bond Graph Modeling

Bond graphs are a domain-independent graphical description of dynamic behavior of physical systems. This means that systems from different domains (electrical, mechanical, hydraulic, and thermodynamic) are described in the same way. The basis is that bond graphs are based on energy and energy exchange.

Bond graph is an explicit graphical tool for capturing the common energy structure of systems. It increases the insight into systems behavior. Moreover, the notations of causality provide a tool not only for formulation of system equations, but also for intuition based discussion of system behavior, viz. controllability, fault diagnosis and observability [2]. In 1959, Prof. H. M. Paynter gave the revolutionary idea of portraying systems in terms of power bonds, connecting elements of the physical system to the so called junction structures which were manifestations of the constraints. This power exchange portray of a system is called Bond Graph (or Bond-graph), which can be both power and information oriented. Later on, Bond Graph theory has been further developed by many researchers like Karnopp, Rosenberg, Thoma and Breedveld, who have worked on extending this modeling technique to power hydraulics, mechatronic, general thermodynamic systems and to electronics and non-energetic systems like economics.
and queuing theory. By this approach, a physical system can be represented by symbols and lines, identifying the power flow paths. The lumped parameter elements of resistance, capacitance and inductance are interconnected in an energy conserving way by bonds and junctions resulting in a network structure. From the pictorial representation of the bond graph, the derivation of system equations is so systematic that it can be algorithmized.

5 Bond Graph Model of BLDC Motor

Typically, a Brushless dc motor is driven by a three-phase inverter with, what is called as a six-step commutation as shown in fig. 4. The conducting interval for each phase is 120° by electrical angle.

![Fig. 4: Simplified equivalent circuit of the BLDC drives](image)

The commutation phase sequence is AB-AC-BC-BA-CA-CB. Each conducting stage is called one step. Therefore, only two phases conduct current at any time, leaving the third phase floating as shown in fig. 5. The commutation timing is determined by the rotor position, which can be detected by Hall sensors. This is why a BLDC motor is also commonly known as an electronically commutated motor (ECM).

![Fig. 5: Typical three phase switching pattern in the BLDC motor](image)

The complete bond graph model of three phase BLDC motor is shown in fig. 6.

![Fig. 6: Bond graph model of BLDC motor](image)

The steps to obtain the differential equations from bond graph follow the answers to two questions:

1. What do the elements give to the system?
2. What do the integrally causaled storage elements receive from the system?

The differential equations for the electrical momentums of three phases, angular momentum and angular displacement from the bond graph (fig. 6) of BLDC motor can be obtained after it is numbered, powered and causaled, as these are the basic inputs necessary to obtain the differential equations. We get the first order differential equations, by integrally causaling each storage elements of bond graph by answering the above two questions.

**Step I :** What do the elements give to the system?

\[
\begin{align*}
\text{Step I:} & \quad \text{What do the elements give to the system?} \\
\frac{df_25}{dt} &= p1/L_a \\
\frac{df_26}{dt} &= p2/L_b \\
\frac{df_27}{dt} &= p3/L_c \\
\frac{df_22}{dt} &= p4/J \\
e_4 &= R_a, f_4 = R_b, f_7 = R_c, f_{25} \\
e_5 &= R_a, f_5 = R_b, f_{26} \\
e_6 &= R_a, f_6 = R_b, f_{27} \\
e_{23} &= R_f, f_23 = R_f, f_{22}
\end{align*}
\]

Where, 
\(f\)’s and \(e\)’s are the flows and efforts in respective bonds of bond graph,
\(p1, p2, p3\) are electrical momentums in three phases;
\(p4\) is the angular momentum of rotor.
\(R_a, R_b, R_c\) are resistances in three phases.
\(L_a, L_b, L_c\) are inductance of three phases.
\(R_f\) is frictional resistance.

Thus the first question is answered for all the elements.

**Step II :** What do the integrally causaled storage elements receive from the system?

\[
\begin{align*}
\frac{dp_1}{dt} &= e_{25} = -e_4 + e_7 - e_{12} \\
\frac{dp_2}{dt} &= e_{26} = -e_5 + e_8 - e_{16} \\
\frac{dp_3}{dt} &= e_{27} = -e_6 + e_9 - e_{20} \\
\frac{dp_4}{dt} &= e_{22} = -e_{23} + e_{11} + e_{15} + e_{19} \\
dq_{24} &= f_{24}
\end{align*}
\]

Thus the second question is answered for all the elements.
Modeling equations for electro-mechanical part of BLDC motor.

\[ e_{11} = f_1(\theta) \cdot K_T, f_{10} \]  
\[ \text{Since, } f_{10} = f_{25}; \text{ (common flow junction i.e.1- Junction)} \]
\[ e_{11} = f_2(\theta) \cdot K_T, f_{25} \]
\[ e_{11} = f_3(\theta) \cdot K_T, p_1/L_a \]  
Similarly,
\[ e_{12} = f_4(\theta) \cdot K_E, f_{13} \]  
But \[ f_{13} = f_{22} = p_4/J; \text{ (common flow junction i.e.1-Junction)} \]
\[ e_{12} = f_5(\theta) \cdot K_E, p_4/J \]  
Where, \( f_a(\theta) \cdot K_T \) = modulus of Gyrator element for torque.
\( f_a(\theta) \cdot K_E \) = modulus of Gyrator element for back emf of phase \( A \)
\( K_E \) = back emf constant.

The state variable \( \theta \) i.e. the rotor position is required to have the function \( f_a(\theta) \), which is given as trapezoidal function:
\[ f_1(\theta) = 1; \text{ if } (0 > \theta > 2.\pi/3) \]
\[ f_2(\theta) = 1-(0-2\pi/3)/6; \text{ if } (2.\pi/3 > \theta > \pi) \]
\[ f_3(\theta) = -1; \text{ if } (\pi > \theta > 5\pi/3) \]
\[ f_4(\theta) = 1+(0-5\pi/3)/6; \text{ if } (5\pi/3 > \theta > 0-2\pi) \]

Similarly, trapezoidal functions for other two phases are:
\[ f_6(\theta) = f_a(\theta - 2 \cdot \pi/3) \]
\[ f_7(\theta) = f_a(\theta - 4 \cdot \pi/3) \]  

6. Simulation of Bond Graph Model

The simulations are done in MATLAB using the default solver ode45. The simulation time has been taken as 0.2 seconds, because beyond 0.2 seconds the characteristics of motor will not be affected.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Moment of Inertia</td>
<td>0.00007</td>
<td>Kg-sq.m.</td>
</tr>
<tr>
<td>Inductance of stator</td>
<td>0.00005</td>
<td>H</td>
</tr>
<tr>
<td>Resistance of stator</td>
<td>1.0</td>
<td>Ohm</td>
</tr>
<tr>
<td>Damping resistance</td>
<td>0.000005</td>
<td>N-m/s/rad</td>
</tr>
<tr>
<td>Torque constant</td>
<td>543</td>
<td>gm-cm/A</td>
</tr>
<tr>
<td>Back EMF constant</td>
<td>6.16</td>
<td>mV/rpm</td>
</tr>
<tr>
<td>Source Voltage</td>
<td>24</td>
<td>volts</td>
</tr>
<tr>
<td>No load current</td>
<td>8.6</td>
<td>A</td>
</tr>
</tbody>
</table>

Fig. 7 shows the phase currents. The current starts with a high value and decays as the motor speeds up until it reaches no load current value, which is about 8 A, which agrees with the value given in parameters above. The current has a nearly perfect quasi-square shape. The only deviation from the quasi-square wave shape occurs at the commutation points. The notches at the commutation points occur because the rise of current in the phase that is being turned on is slower than the decay of the current in the phase that is being turned off. The notches are the cause of the torque ripples of BLDC motor.

It is clear from the fig. 8 that the current in phase A is saturated after some time as explained in fig. 7. After that sudden rise and sudden drop in current is due to inverter switching. Also the shape of back EMF wave is trapezoidal. This shape is essential in BLDC motor in order to obtain linear relationship between torque produced by the rotor and the current. The angular displacement is continuously increasing, this confirms that torque is continuously produced and the concept of modulated transformer (“MTF”), as used in bond graph for extracting information about switching sequence is correct.

The plot of total torque with respect to angular displacement is shown in Fig. 9. We observe from the Fig. that, initially the torque is higher. This is due to higher current at the beginning. As soon as the motor attains constant speed, the torque is stabilized.

Fig. 7: Current in three phases with respect to time

Fig. 8: Current in phase A, Back EMF of phase A and angular displacement of rotor.

Fig. 9: Plot of torque v/s angular displacement.
The appropriateness of bond graph model of motor is evident from the various plots as obtained during simulation.

7 Conclusions

The applications of BLDC motors and drives have grown significantly in recent years in the appliance industry and the automotive industry. The fast growth of the BLDC motor in applications requiring positional and speed accuracy has increased the need for optimum and precise modeling of their dynamics and drive circuit. Modeling of armature circuit and dynamics of BLDC motor have been the focus of this paper.

As a matter of fact the conventional modeling methods involve more computational work. The aim of this paper was to make a model that would be simple, accurate, easy to modify and suitable for real time implementation. The simulation results of this work have shown that these goals have been achieved. In this paper, a unified approach of bond graph for modeling of three phase BLDC motor and its drive circuit was used, analyzed, and extended, for overcoming the drawbacks of the conventional modeling approaches.

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