

# On Simplified Solutions in Tribological Design of Machine Components

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## Abstract

The paper will highlight some simplified treatments of tribological problems. It is shown that these solutions provide reasonable accuracy in designing machine components. However, designers and users should be aware of their limitations.

**Keywords:** Lubrication, fluid film bearings

## 1 Introduction

Just Report defines *Tribology* is “the science and technology of interacting surfaces and of related practices thereof”. However, the subject generally deals with i) the technology of lubrication, ii) friction control and iii) wear prevention. A concentrated effort is directed towards the methods of controlling friction and minimizing wear of machinery by: a) adopting new lubrication technology, b) developing new lubricants and c) finding new wear resistant materials. Two major constituents of tribology, friction and wear, when controlled and reduced, automatically increase the service life of machinery. This, in turn, saves money. The awareness of the subject, the identification of tribological problems and their solutions can give rise to significant savings. For this reason government, research & development organisations and many industries are placing emphasis on the economic aspect of tribology.

In this note a few simplified solutions which provide reasonably accurate results are discussed. Although most of the materials given here are available in the open literature, their use and application to the design of fluid film bearings can avoid a lot of mathematical complexity. Moreover, the inherent simplicity of the approach is attractive.

## 1.1 Bearings

A bearing is a system of machine components whose function is to support load (radial or axial) by reducing friction between the relatively moving surfaces.

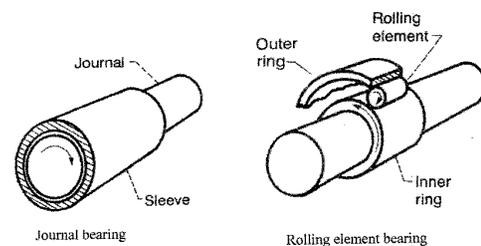


Fig.1 Bearings

Two most widely used bearings (Fig.1) are:

- Fluid film bearings
- Rolling-element bearings

## 1.2 Bearing Selection

Rolling-element bearings have much wider use in industries since rolling friction is lower than the sliding friction. Lubricant used in these bearings is usually grease. Fluid film bearings are lubricated by the hydrodynamic flow which is generated by relative surface motion/external pressurization. Here the load supporting ability is due to wedging effect of the fluid caused by relative tangential motion between two surfaces.

Rolling-element bearings are more desirable than fluid film bearings in situations,

- the ability to support combined radial and thrust loads.
- less sensitive to interruption of lubrication.
- the ability to seal the lubricant with the bearing.

The following situations make rolling-element bearings

less desirable,

- finite fatigue life
- low damping capability
- high noise level
- more severe alignment

### 1.3 Background

The original contribution to the theory of lubrication came out from the work of Tower (1883) in England and Petroff (1883) in Russia. Tower while conducting experiments on partial oil journal bearings used in railways car wheels observed the development hydrodynamic pressure first. Petroff applied Newton's law of viscous flow and expressed the coefficient of fluid film friction. Three years later Reynolds (1886) derived basic equation in the theory of lubrication based on the experiments of Tower. Tower's experimental results were analysed and interpreted. Although Reynolds derived the governing equation in 1886, the first solution assuming the journal bearing as an infinitely long was given by Sommerfeld in 1904. Considering bearing to be infinitely short (narrow), Ocvirk gave an analytical solution in 1952. The solution obtained using narrow bearing theory gives better approximation. Pinkus (1958) and Raimondi and Boyd (1958) gave numerical solution of real (finite) oil journal bearings using digital computer. Later many researchers including the author (1981) provided numerical solutions using cavitation boundary conditions for various bearing configurations.

### 1.4 Regimes of Lubrication

While dealing with the theory of lubrication it is sometimes necessary to understand the variation of coefficient of friction with film thickness in the different regimes of lubrication, especially in the elastohydrodynamic and hydrodynamic regimes. Fig.2 shows four regimes, boundary, partial, elastohydrodynamic and hydrodynamic of lubrication.

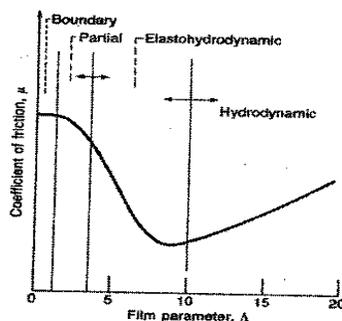


Fig.2 Regimes of lubrication

The coefficient of friction  $\mu$  is defined as

$$\mu = F/W \quad (1)$$

In Fig.2, the film parameter  $\Lambda$  is defined as:

$$\Lambda = \frac{h_{min}}{\sqrt{\sigma_a^2 + \sigma_b^2}} \quad (2)$$

where  $\sigma_a$  and  $\sigma_b$  are the standard deviations of surfaces a and b. In hydrodynamic lubrication,  $5 \leq \Lambda \leq 100$ . As the film parameter increases, coefficient of friction decreases in the elastohydrodynamic regime and then increases in the hydrodynamic regime. In hydrodynamic lubrication of conformal surfaces as found in journal and thrust bearings  $W \propto \frac{1}{h}$ . In elastohydrodynamic lubrication of non-conformal surfaces the applied load has little effect on film thickness. Therefore W is nearly constant. In both hydrodynamic and elastohydrodynamic lubrication friction force F can be expressed as

$$F \propto \frac{1}{h}$$

Hence ,

$$\mu_{HL} \propto \left(\frac{1}{h}\right) \propto h \quad (3)$$

$$\mu_{EHL} \propto \frac{1}{\text{constant}} \propto \frac{1}{h}$$

This explains the reverse trend of coefficient of friction. Fig.2 gives a clear picture of variation of coefficient of friction with film thickness when  $\sigma_a$  and  $\sigma_b$  are kept constant.

## 2 Simplified Solutions:

As already mentioned some simplified solutions in the theory of lubrication that give reasonably accurate results are outlined.

### 2.1 Petroff Equation

In 1883 Petroff calculated the friction force and coefficient of friction in a lightly loaded oil journal bearing (that is, the journal is running almost concentrically with bearing). Petroff calculated the friction variable as

$$\left(\frac{\mu}{c}\right) \mu = 2\pi^2 S \quad (4)$$

here S is Sommerfeld number and given by  $S = \frac{4\pi N}{P} \left(\frac{r}{c}\right)^2$  where  $P = \frac{W}{LD}$

Equation (4) is called the Petroff equation. It shows that coefficient of friction for a particular bearing is inversely proportional to load and directly proportional to journal speed. If we calculate the friction variable for an infinitely short journal bearing using Ocvirk's narrow bearing theory, it will be found as

$$\left(\frac{R}{C}\right) \mu = \frac{\epsilon \pi^2 S}{\sqrt{(1-\epsilon^2)}} \quad (5)$$

When  $\epsilon \rightarrow 0$  (a lightly loaded bearing), Equations (4) and (5) are identical. Petroff used simple mechanics to obtain Equation (4), whereas Ocvirk(1952) solved one dimensional Reynolds equation satisfying the appropriate boundary conditions using narrow bearing theory for Equation(5). It may be noted that Equation(4) does not satisfy fluid dynamics equation. It cannot be used for moderately and heavily loaded bearings as radial clearance has been assumed to be constant in the derivation. But its inherent simplicity is attractive and it is still considered to be the classical work in the theory of lubrication.

## 2.2 Oil whirl

Rotating machines are supported in fluid film bearings. These bearings provide relatively low coefficient of friction and good damping. One of the most troublesome problems of high speed journal bearings is oil whirl. The journal may whirl in translatory or conical mode (Fig.3(a)). The shaft will whirl in one of the modes depending on the span and other geometrical characteristics. By conventional methods (linearized and non-linear transient) (2005) the journal speed at the threshold of instability can be determined. This requires complex mathematical exercise by solving the Reynolds equation under dynamic condition for stiffness and damping coefficients of fluid film and equations of motion of the shaft supported on these bearings for stability (mass parameter and whirl ratio). The whirl ratio (the ratio of speed of journal centre to journal speed) comes out to be nearly equal to  $\frac{1}{2}$  (Fig.3(b)). That is why it is also called 'half-frequency whirl' or 'half-speed' whirl.

The ratio can be easily found from the continuity of flow in the gaps at the minimum and maximum film thicknesses (Fig.3(b)).

In the gap 10 the oil flow is:  $\left(\frac{R\omega}{2}\right)(c - e)$

In the gap 02 the oil flow is:  $\left(\frac{R\omega}{2}\right)(c + e)$

At the instant shown in the above figure, the shaft is moving vertically upwards with an instantaneous velocity. From continuity consideration

$$\left(\frac{R\omega}{2}\right)(c - e) + 2R\omega_p e - \left(\frac{R\omega}{2}\right)(c + e) = 0$$

Or  
 $R \cdot e \cdot (2\omega_p - \omega) = 0$   
 (6)

As  $R \cdot e \neq 0$ , hence,  $2\omega_p = \omega$  or  $\frac{\omega_p}{\omega} = \frac{1}{2}$

From a rigorous analysis (the first-order perturbation

method) made by the author(2005) the calculated values of the whirl ratio of two journal bearings given in Table 1 are 0.4735 and 0.4687. These are very close to 0.5 as obtained from the simplified analysis. However, the simplified analysis does not provide mass parameter, a measure of threshold parameter. In that sense the analysis is not complete. Nevertheless, the analysis considering flow at two film thicknesses is very simple.

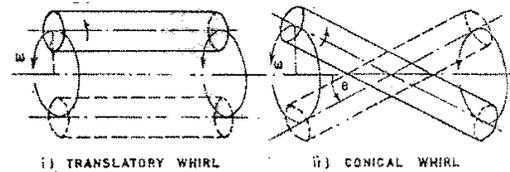


Fig.3a Rotating journal bearing.

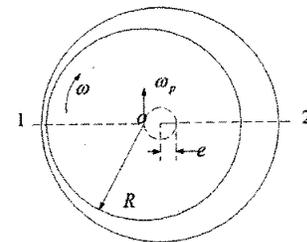


Fig.3b Rotating journal bearing.

Parameter	Bearing 1	Bearing2
Shaft Diameter, D	100 mm	200 mm
Shaft Speed, N	3500 rpm	30000 rpm
Radial Load, W	250kN	1000kN
Rotor Mass/ Bearing, M	7.5 kg	15 kg

Table 1: Details of operating parameters

From consideration of space and other limitations, the length to diameter ratios of Bearings 1 and 2 are taken as 1 and 0.6, respectively. Radial clearances (C) for Bearings 1 and 2 are 0.025 mm and 0.05 mm. The rigid shaft is supported on two identical journal bearings. The analysis that gives mass parameter and whirl ratio of the above bearings are:

Bearing 1: Mass parameter = 7.6040, Whirl ratio = 0.4735

Bearing 2 : Mass parameter = 7.0836, Whirl ratio = 0.4687.

Following non-linear transient analysis (1995) which provides orbit details, the mass parameters (7.6 and 7.0) for the above two bearings are found and journal centre trajectories are shown in the form of polar plots (Fig.4).

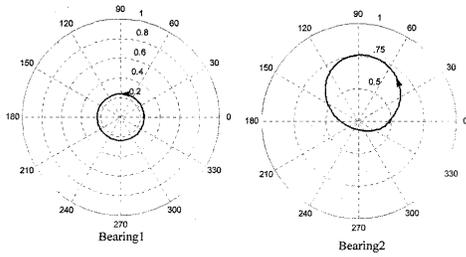


Fig.4 Journal centre trajectories.

### 2.3 Linear pressure drop approximation

In this section a couple of problems making use of linear pressure drop approximation in the land area are taken up.

#### 2.3.1 Oil flow through a journal bearing having circumferential oil groove at the mid plane.

If the direction of the radial load on a full journal bearing varies during the operation, such as an i.e. engine bearing, it impossible to locate inlet oil hole or groove in the unloaded region of the bearing. Under this situation there two possibilities of feeding system: submerged bearings and circumferential oil groove for feeding oil. A detailed analysis has been given by the author and his co-workers (1995) for submerged bearings. Here the second possibility i.e. journal bearing having circumferential oil groove is considered.

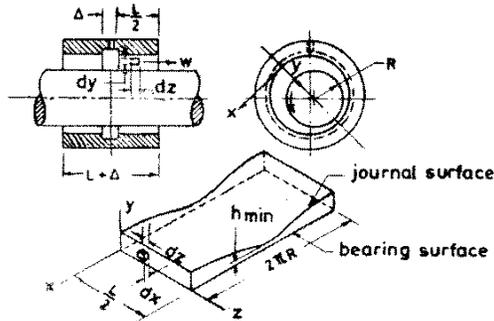


Fig.5 A journal bearing with circumferential oil groove

It can be shown easily that the volume rate of oil flow from one end of bearing is:

$$Q = \frac{\pi R C^3}{6\eta} \frac{\partial p}{\partial z} (1 + 1.5\epsilon^2) \quad (7)$$

If one makes linear pressure drop assumption from mid plane to the end of the bearing,

$$\frac{\partial p}{\partial z} = \frac{p_0}{L} \quad (8)$$

where  $p_0$  = feed pressure,  $L$  = bearing length

Substitution of  $\frac{\partial p}{\partial z}$  in equation (7) gives Q as from one end.

$$Q = \frac{\pi R C^3 p_0}{6\eta L} (1 + 1.5\epsilon^2) \quad (9)$$

As the flow occurs from both the ends, the total outflow from the bearing ends is

$$Q = \frac{2\pi R C^3 p_0}{6\eta L} (1 + 1.5\epsilon^2) \quad (10)$$

Although the flow rate given in equation (10) is approximate since it does not consider the effect of hydrodynamic pressure due to rotation, it is fairly accurate at low eccentricity ratios and small speeds.

#### 2.3.2 Externally pressurized circular step gas thrust bearings.

Referring to Fig. 6 the pressure distribution in land is given by

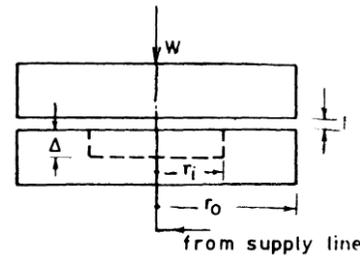


Fig. 6 A circular externally pressurized gas thrust bearing.

$$p = p_0 \left[ 1 - \frac{1 - \left(\frac{2z}{\Delta}\right)^2}{\ln\left(\frac{r_o}{r_i}\right)} \ln\left(\frac{r}{r_i}\right) \right]^{\frac{1}{2}} \quad (11)$$

The load capacity W is

$$W = \pi p_0 \eta^2 - \pi p_0 p_0^2 + \int_{r_i}^{r_o} 2\pi r p dr \quad (12)$$

The integral  $2\pi \int_{r_i}^{r_o} r p dr$  is difficult to evaluate.

If the linear pressures drop assumption in the land area is used

$$p = p_r - p_a \quad 0 \leq r \leq \eta \quad (12a)$$

$$p = (p_r - p_a) \left( \frac{r_o - r}{r_o - r_i} \right) \quad \eta \leq r \leq r_o \quad (12b)$$

Introducing p in the integral, the load capacity will come out as

$$W = \frac{F}{4} (p_1 - p_2) \left( \frac{r_2^2 - r_1^2}{r_2 - r_1} \right) \quad (13)$$

Self-excited vibrations of air-lubricated thrust bearing,  
*Trans. ASME*,80.

The expression in equation (13) is fairly accurate. Moreover, the use of equation (13) in the analysis under dynamic condition (pneumatic instability) (1958) can avoid a lot of mathematical complexity.

## 5 Conclusions

The foregoing report outlines a few fairly accurate simple rules can guide designers and practising engineers in designing fluid film bearings, which are vital components in machinery.

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