

# Analysis of Dynamic Response of the Implement Lift in the Low Horsepower Mini Tractor

Abhijit Mahapatra\*, Avik Chatterjee  
Virtual Prototyping & Immersive Visualization Lab,  
Central Mechanical Engineering Research Institute,  
M.G.Avenue, Durgapur 713209, (CSIR), India.

\*Corresponding author (email: abhi\_mahapatra@yahoo.co.in)

## Abstract

A Full 1:1 scale digital model of the tractor was designed and some critical functional issues like the implement lift were simulated which helped to catch problems early in the design stage and to reduce multiple physical prototypes for testing and validation. To reduce complexity, the spatial model of the Tractor implement has been simplified to a planar mechanism of eight-bar linkage to develop the analytical model. Rigid Body Dynamic models have been developed based on the Euler Lagrangian formulation. The paper describes the use of the commercial software code to investigate and iterate the complex actuation methods for lifting the implement with time as well as the behavior of the aggregate center of mass (CM) with the implement lift which is very important in stability analysis of the entire system.

**Keywords:** Multibody Dynamics, Tractor Implement, Simulation, Aggregate CM, Dynamic Modeling

## 1 Introduction

Today the new product development process makes use of both virtual and physical prototypes. Virtual Prototyping (VP) enables quick product development and adds quality to the development process, enabling system-based decisions. Therefore, convergence of technologies such as simulation, computer aided design (CAD) and Virtual Reality (have enabled the development of accessible, low cost, user-friendly VP systems. These VP tools are increasingly being viewed as the next generation of computerized design systems. In this exercise, it is targeted to model the mechanical system of the system in its true representation to simulate and visualize its 3D – motion behavior under

real world operating conditions, refining/ optimizing the design through iterative design studies prior to building the first physical prototype.

In conventional modeling and simulation a reduced representation of a mechanical system is assumed and then its function is simulated using various concepts of Science & Engineering. But the present paper describes the development of the full 1:1 scale CAD model of the low horsepower mini tractor and subsequently carrying out dynamic simulation of the system in Automatic Dynamic Analysis of Mechanical Systems (ADAMS) solver, thereby, helping to catch problems early in the design stage and reducing multiple physical prototypes for testing and validation. Moreover using ADAMS [1-4], one can solve very complex problems which are very difficult to solve analytically. The time savings over the manual approach are also very significant.

In India there has been a huge demand for developing small, compact and easily maneuverable tractors of ratings of low Horsepower (HP) i.e.12 or less, which are deemed fittest for the small and fragmented land holdings. Therefore, for affording farm mechanization and increasing the yield, CMERI undertook the development of a small 12 HP mini tractor. The project undertaken at CMERI aimed at development of the small tractor and its matching implements. To select and match tractors and implements, one will need information about the capacity of the tractor and implement as well as the load that is likely to be imposed on the power unit.

This paper presents a comprehensive and realistic dynamic model of a tractor-implement combination. Full scale functional VP simulation of the implement was carried out to check some of the critical functional issues like behavior of the implement due to action force which might be variable or constant. Hence it is possible to eliminate the majority of guess work that is normally employed when a machinery purchase decision has to be made.

## 2 Modeling of the Tractor with

## Matching Implement

Although ADAMS [5] is a very powerful simulation and design software, its solid modeling ability is weaker than commercial CAD software. However, sophisticated 3-D geometry information can be readily imported into ADAMS from many commercial CAD tools such as IDEAS, CATIA V5, Unigraphics, ProEngineer, etc. In the present work, a three dimensional CAD model [6] of Mini Tractor has been developed in IDEAS. Parts of model were then exported in .iges file format and imported into Computer Aided Three Dimensional Interactive Application (CATIA) for assembling as shown in Fig. (1). Thereafter, necessary joints and contacts were defined in CATIA V5 SimDesigner r13 and exported as a .cmd file along with .sh files. The .cmd file is readily imported into ADAMS. The three main steps followed to develop virtual prototype of the tractor are given below.

Step 1: Part modeling of the different parts of the robot in IDEAS

Step 2: Assembling of the parts in CATIA V5

Step 3: Defining of joints and contacts for the robot in CATIA SimDesigner and exporting it into ADAMS for dynamic simulation.

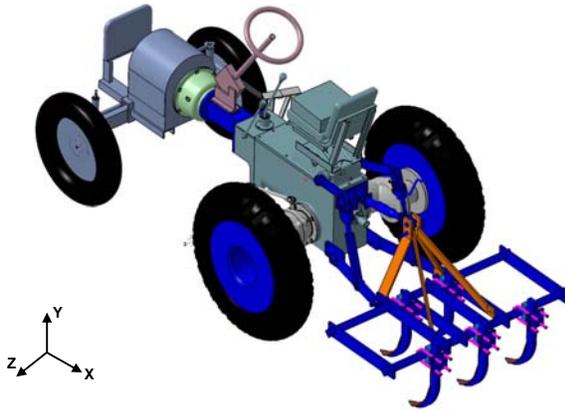


Fig. 1 Mini Tractor

Total mass of the matching implement as obtained from the model is 125.0 Kg (approximately).

## 3 Rigid Multibody Dynamic Simulation of the Implement lift

In order to develop more accurate dynamic model, computer aided simulation tools based on rigid multibody dynamics called ADAMS has been used. The tractor with matching implement was simulated in ADAMS/Solver [7]. The following assumptions are made about the tractor to simplify the rigid multibody dynamic analysis.

- i. Tractor is stationary while the implement is lifted.

- ii. Ground reaction on the digging implement is replaced with equal resultant force acting on the knife edges.
- iii. At the highest position of the implement angle between the connecting arm (crank) angle and the vertical axis is  $45^{\circ}$ .
- iv. At the lowest position (taking into account the dig on the ground) of the implement angle between the connecting arm (crank) angle and the vertical axis is  $-20^{\circ}$ .

Some of the important data provided by the designer are as shown in Table 1.

Table 1: Designer's data

Parameter	Value
Connecting Rod length	134 mm
Hydraulic Cylinder Pressure	160 bar
Rock Shaft Diameter	40mm
Piston diameter	50mm
Piston Area	1962.5mm <sup>2</sup>
Hydraulic thrust	31400 N
Arm Radius	75 mm
Moment	1177500 N-mm
Implement depth	4inch
Draw bar pull	400 kg.
Angle between Connecting rod and Rocker arm at lower position	110 deg.

In the present work, the implement has been simulated by applying an actuated force on the piston (equivalent to hydraulic thrust) and its affect on the motion behavior of the implement during operation have been studied. The following two aspects with respect to dynamic stability of implement has been checked for,

- (a) Implement motion due to constant force
- (b) Implement motion due to regulated force

ADAMS uses Lagrange's equation of the first kind [8], shown in Eq. 1, to form the differential equations of motion of the model automatically.

$$\frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}_i} \right) - \frac{\partial L}{\partial q_i} = \sum_{j=1}^m \lambda_j a_{ji} \quad (1)$$

where the Lagrangian function  $L = T - V$ .  $T$  represents the kinetic energy;  $V$  represents the potential energy;  $q_i$  is the  $i^{\text{th}}$  generalized coordinate;  $\lambda_j$  is the Lagrange multiplier;  $a_{ji}$  is the derivative of the constraint equation [9].

Motions for force actuation through the piston are defined by STEP MATH FUNCTION as shown in Fig.(2). The simulated implement is evaluated for 60 time steps. In the simulation, each time step represents an integration step in which the new positions, orientations, velocities and acceleration of the implement parts are computed based on the forces acting on them.

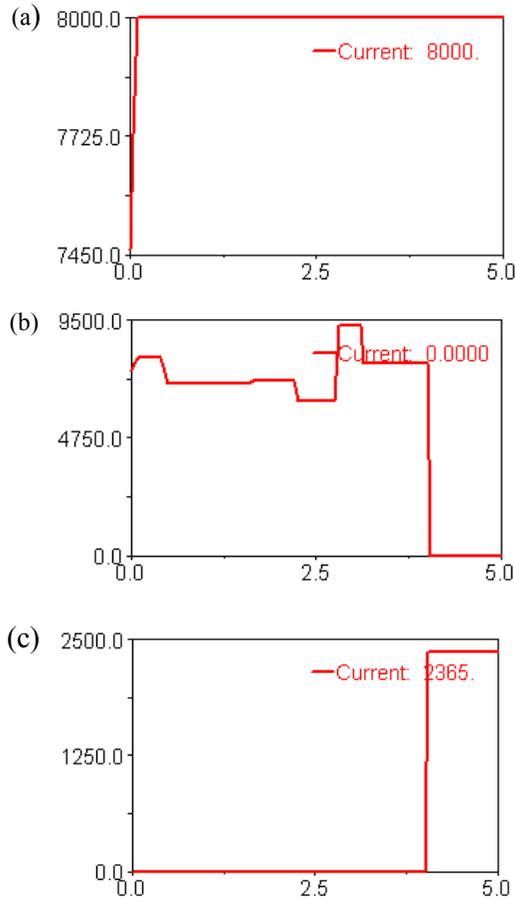


Fig. 2: STEP MATH FUNCTION (a) Constant force (b) Regulated Force: *Action Force by the piston* (c) *Reaction Force due to implement digging into the ground*:

To reduce complexity, the spatial model of the Tractor Implement has been simplified to a planar mechanism of eight-bar linkage and the theoretical model was developed for the study. Dynamic models have been developed based on the kinematics and dynamics of machinery and the Euler Lagrangian dynamic formulation. Refer Appendix.

## 4 Results & Discussion

Simulations have been carried out to investigate the effect of actuated constant force against that of actuated regulated force. Also the behavior of the aggregate CM with implement lift has been studied.

Fig. (3) & (4) shows the results obtained from ADAMS post processor both in the case of actuated constant force and actuated regulated force i.e. variation of the lifting velocity of the implement and variation of the connecting arm angle w.r.t. time respectively.

The application of lateral disturbances (such as erratic velocity of the implement as shown in Fig. (4a),

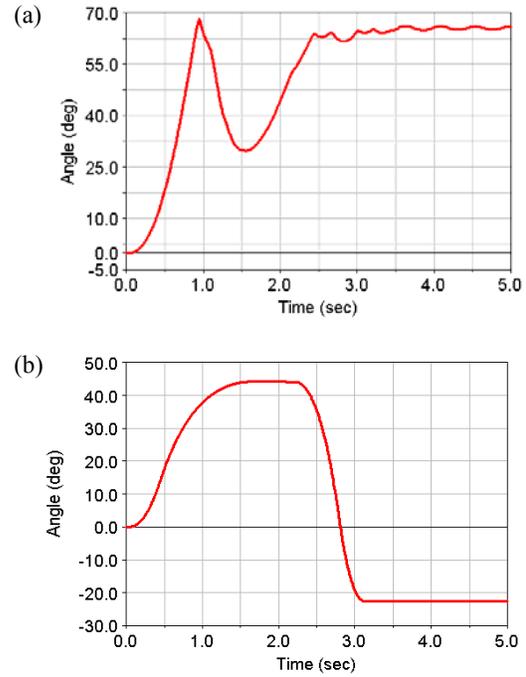


Fig. 3: Variation of connecting arm angle w.r.t Y-axis (a) actuation by constant force (b) actuation by regulated force

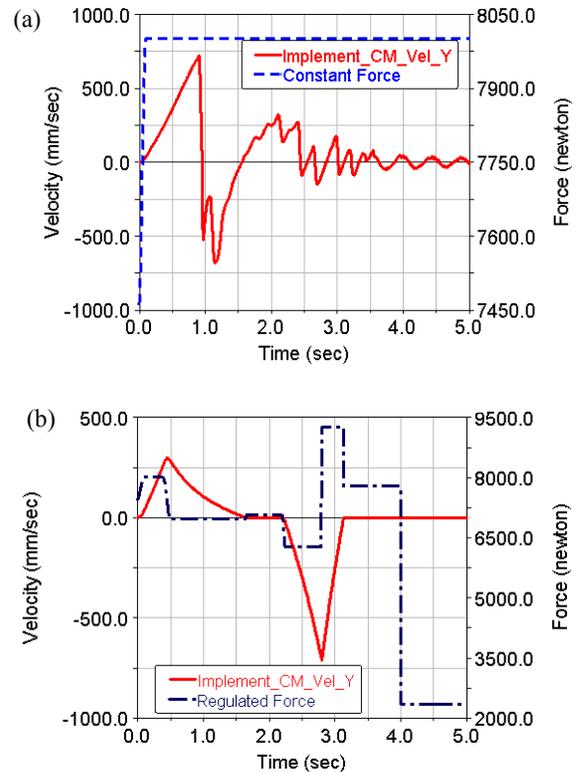


Fig. 4: Change in the velocity of the implement w.r.t time (a) actuation by constant force (b) actuation by regulated force.

joint forces etc.) in case of constantly actuated force underlines the requirement that actuation be sufficiently robust in order to maintain implement on its desired paths. It is to be remembered that such lateral disturbances are particularly relevant in an agricultural or precision farming environment, where ground undulation and sloping terrain may play a role. So the simulation was carried out by regular force actuation to study its affect on the implement. Results show the implement to move on its desired path and also minimization of the lateral disturbances (see Fig. 4b).

Also it is very essential to determine the position of the aggregate CM at any instant of time during motion of the implement to check its dynamic stability. The variation of aggregate CM can be well investigated in

ADAMS. For that user written sub-routine was run. Firstly, the user written subroutine centroid.cmd file was imported to the ADAMS workbench and simulation was carried out. Finally, to visualize the variation of the aggregate CM at any instant of time, the user subroutine AGG\_CM\_STATE\_VARIABLE\_IMPORT.cmd was imported.

Simulation shows that due to application of constant force the CM behavior of the implement is irratic and hence destabilizes the implement. But when a regulated force is applied there is no such off bit path variation of the CM and the trace is a smooth curve (Fig.5). The implement is dynamically stable. Some of the results obtained from the simulation, actuated by regulated force are tabulated below (see Table -2).

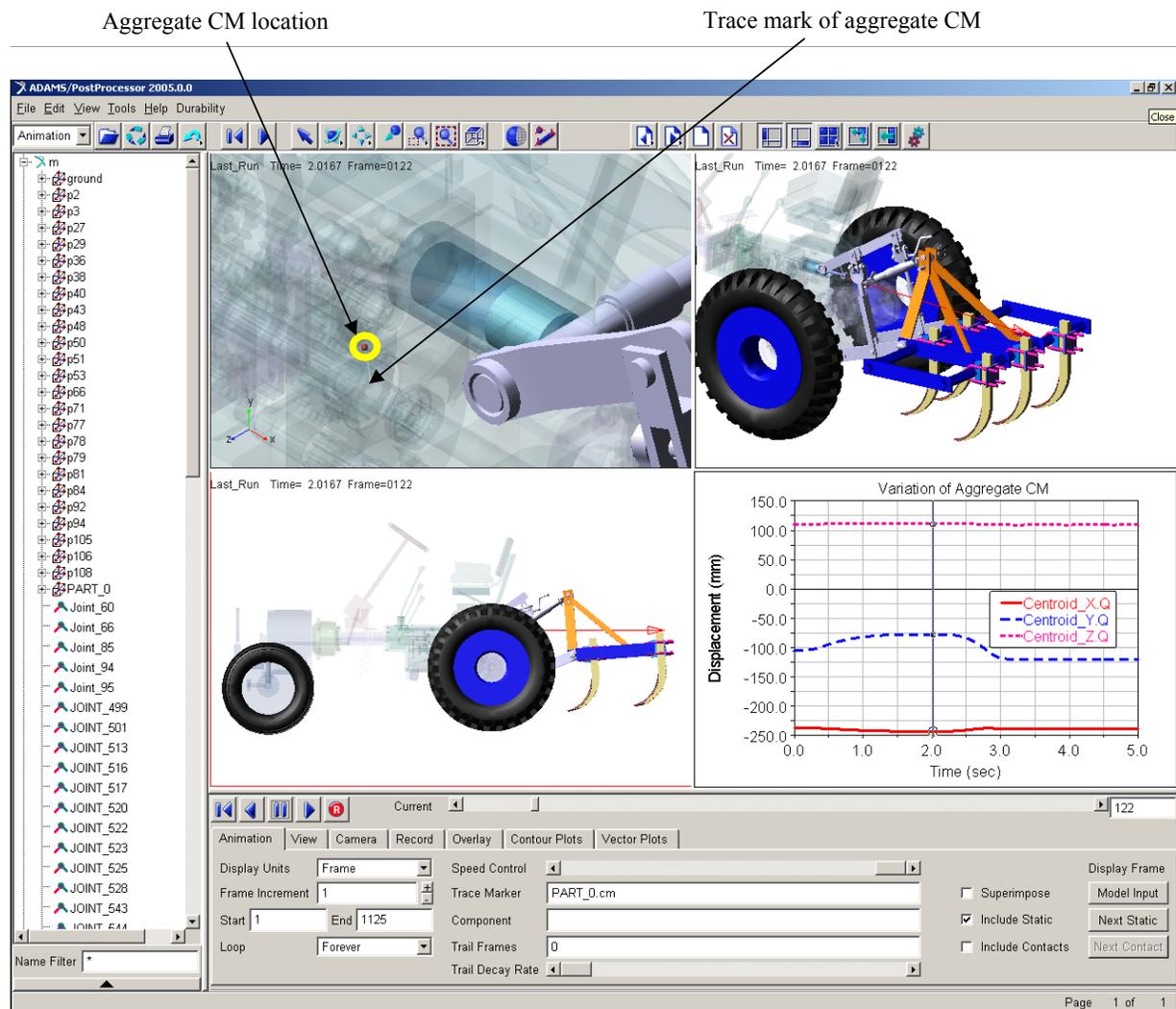


Fig. 5: Aggregate CM variation of tractor implement during simulation

**Table-2:** Implement simulation results

Parameter	Value
• Equilibrium angle due to crank rotation from vertical position	
-Implement at top position	44.20 deg
-Implement at bottom position	22.70 deg
• Angle between Connecting rod and rocker arm at lower position of the implement	113.0 deg.
• Equilibrium Force (taken along +ve x-axis)	
-Implement horizontal with x-axis	7462.5N
-Implement at top position	7052.0N
-Implement at bottom position	7780.0N
• Implement depth	102.1 mm or 4.02 inch
• Ground reaction force (at 4.02 inch depth)	2365N

## 5 Conclusion

Simulation showed the significant effect that actuated force has on the tractor implement. The study helped to determine the variation of the force with the lifting velocity of the implement required for the design. The importance of this phenomenon cannot be understated as it can be a significant problem in a precision farming setting. Importantly, the CAD model, and the results from the ADAMS simulation, provides a sound basis for which to continue research into more precision implement guidance. Further it will be necessary to carry out experimental model validation to confirm these results.

## Appendix

Kinematic constraints that describe mechanical joints as well as specified trajectories in the multibody system consisting of interconnected rigid components can be formulated by using a set of non-linear algebraic constraint equations i.e.

$$\mathbf{C}(q_1, q_2, \dots, q_n, t) = \mathbf{C}(\mathbf{q}, t) = \mathbf{0} \quad \text{A.1}$$

where  $\mathbf{C} = [\mathbf{C}_1(\mathbf{q}, t) \mathbf{C}_2(\mathbf{q}, t) \dots \mathbf{C}_m(\mathbf{q}, t)]^T$  is the set of  $m$  independent constraint equations,  $t$  is the time, and  $\mathbf{q}$  is the total vector of system generalized coordinates given by  $\mathbf{q} = [q_1 \ q_2 \ q_3 \ \dots \ q_n]^T$ ,  $n$  the number of generalized coordinates such that  $m \leq n$ . Differentiating Eq. (A.1) with respect to time yields,

$$\mathbf{C}_q \dot{\mathbf{q}} + \mathbf{C}_t = \mathbf{0} \quad \text{A.2}$$

For a constraint rigid multibody dynamic system with  $n$  bodies and  $m$  (holonomic and non-holonomic) constraint equations, Eq. (A.2) can be written as,

$$\sum_{k=1}^n C_{jk} \dot{q}_k + C_{jt} = 0, j = 1, 2, \dots, m \quad \text{A.3}$$

$$\text{where } \mathbf{C}_q = \begin{bmatrix} C_{11} & C_{12} & \dots & C_{1n} \\ C_{21} & C_{22} & \dots & C_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ C_{m1} & C_{m2} & \dots & C_{mn} \end{bmatrix},$$

$$\mathbf{C}_t = [C_{1t} \ C_{2t} \ \dots \ C_{mt}]$$

$$\text{NB: } C_{jk} = \frac{\partial \dot{x}_j}{\partial q_k}, \quad C_{jt} = \frac{\partial \dot{x}_j}{\partial t}$$

Differentiating Eq. (A.3) w.r.t time,

$$\mathbf{C}_q \ddot{\mathbf{q}} + \dot{\mathbf{C}}_q \dot{\mathbf{q}} + \dot{\mathbf{C}}_t = \mathbf{0} \quad \text{(A.4)}$$

$$\text{or } \mathbf{C}_q \ddot{\mathbf{q}} = -\dot{\mathbf{C}}_q \dot{\mathbf{q}} - \dot{\mathbf{C}}_t = \mathbf{Q}_c \quad \text{(A.5)}$$

where  $\mathbf{Q}_c$  is a function of  $\mathbf{q}, \dot{\mathbf{q}} \& t$ .  $\mathbf{Q}_c$  is a vector that depends on the reference coordinates and velocities and possibly on time.  $\mathbf{C}_q$  is the constraint jacobian matrix.

Lagrange's equations of motion for a rigid multibody system are

$$\frac{d}{dt} \left( \frac{\partial T}{\partial \dot{q}_k} \right) - \frac{\partial T}{\partial q_k} = Q_k + \sum_{j=1}^m \lambda_j C_{jk} \quad k = 1, \dots, n \quad \text{(A.6)}$$

where  $Q_k$  is the generalized force corresponding to generalized coordinate  $q_k$ ,  $\lambda_j$  is the Lagrange Multiplier associated with  $j^{\text{th}}$  constraint (the Lagrange Multipliers are related to the forces and moments required to maintain the constraints),  $C_{jk}$  are the coefficient from the constrained equations.

In matrix form, the general system differential equation of motion of the rigid multibody system can be written as,

$$\mathbf{M} \ddot{\mathbf{q}} - \mathbf{C}_q^T \boldsymbol{\lambda} = \mathbf{Q}_e + \mathbf{Q}_v \quad \text{(A.7)}$$

where  $\mathbf{Q}_e$  and  $\mathbf{Q}_v$  are functions of  $\mathbf{q}, \dot{\mathbf{q}} \& t$ .  $\mathbf{M}$  is the mass matrix of the system,  $\mathbf{Q}_e$  is the vector of generalized externally applied forces, and  $\mathbf{Q}_v$  is the quadratic velocity vector that contains the gyroscopic as well as Coriolis components and results from differentiating the kinetic energy w.r.t. time and w.r.t. to the system generalized coordinates  $\mathbf{q}$  and  $\boldsymbol{\lambda}$  is the vector of Lagrange multipliers. Combining Eq. (A.5) and Eq. (A.7) we get,

$$\begin{bmatrix} \mathbf{M} & \mathbf{C}_q^T \\ \mathbf{C}_q & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{q}} \\ -\boldsymbol{\lambda} \end{Bmatrix} = \begin{Bmatrix} \mathbf{Q}_e + \mathbf{Q}_v \\ \mathbf{Q}_c \end{Bmatrix} \quad \text{(A.8)}$$

Therefore, Eq. (A.8) provides a set of  $n+m$  algebraic equations for the  $n+m$  unknowns i.e. the  $n$  system coordinates and  $m$  Lagrange Multipliers  $\lambda_j$  ( $j=1, \dots, m$ ). The equations are differential in the generalized coordinates and algebraic in the Lagrange Multipliers. For an example, modeling of "Mini Tractor Implement" is shown below.

Here we present a complete mathematical model that takes in to account the dynamics of a tractor implement. The implement has been simplified to a planar mechanism of 8 bar linkage.

The tractor implement is considered as a rigid multibody system which has the global fixed inertial frame  $\{E\}$  represented by X-Y coordinate system (fixed at O) & the body fixed (here links) frame  $\{B\}$  represented by  $\xi$ - $\eta$  coordinate system as shown in Fig. (6). The origin of the body fixed coordinate frame is fixed at the centre of gravity (CG) of the trunk. The number of degrees of freedom (DOF) for this mechanism is  $k=8 \times 3 - (9 \times 2 + 1 \times 2 + 3) = 1$ , since there are  $8 \times 3 = 24$  coordinates in the system, 9 revolute joints eliminate 18 DOF, 1 translational joint eliminate 2 DOF, and ground constraints on body 1 eliminate 3 DOF.

The constraint equations for the revolute joints are obtained from the vector loop Eq. (A.9) as illustrated in Fig. (7).

$$\begin{aligned} \mathbf{r}_2 + \mathbf{s}_2^N - \mathbf{r}_3 - \mathbf{s}_3^N &= \mathbf{0} \\ \mathbf{r}_3 + \mathbf{s}_3^M - \mathbf{r}_4 - \mathbf{s}_4^M &= \mathbf{0} \\ \mathbf{r}_2 + \mathbf{s}_2^P - \mathbf{r}_5 - \mathbf{s}_5^P &= \mathbf{0} \\ \mathbf{r}_5 + \mathbf{s}_5^Q - \mathbf{r}_6 - \mathbf{s}_6^Q &= \mathbf{0} \\ \mathbf{r}_6 + \mathbf{s}_6^S - \mathbf{r}_7 - \mathbf{s}_7^S &= \mathbf{0} \\ \mathbf{r}_7 + \mathbf{s}_7^T - \mathbf{r}_8 - \mathbf{s}_8^T &= \mathbf{0} \end{aligned} \quad (\text{A.9})$$

System kinematics constraint equations i.e.  $m=24$  were developed manually as written below. See Eq. (A.10)

Ground Constraints:

$$\begin{aligned} \Phi_1 &\equiv x_1 = 0.0 \\ \Phi_2 &\equiv y_1 = 0.0 \\ \Phi_3 &\equiv \theta_1 = 0.0 \end{aligned}$$

Revolute Constraints:

$$\begin{aligned} \Phi_4 &\equiv x_2 - R_0 \sin(\alpha_1 + \theta_2) - x_1 = 0 \\ \Phi_5 &\equiv y_2 + R_0 \cos(\alpha_1 + \theta_2) - y_1 = 0 \\ \Phi_6 &\equiv x_3 + R_{23} \sin \theta_3 - x_2 + R_{32} \sin(\alpha_2 + \theta_2) = 0 \\ \Phi_7 &\equiv y_3 - R_{23} \cos \theta_3 - y_2 - R_{32} \cos(\alpha_2 + \theta_2) = 0 \\ \Phi_8 &\equiv x_4 + R_{34} - x_3 + R_{43} \sin \theta_3 = 0 \\ \Phi_9 &\equiv y_4 - y_3 - R_{43} \cos \theta_3 = 0 \\ \Phi_{10} &\equiv x_5 - R_{25} \sin \theta_5 - x_2 - R_{52} \sin(\alpha_3 + \theta_2) = 0 \\ \Phi_{11} &\equiv y_5 + R_{25} \cos \theta_5 - y_2 + R_{52} \cos(\alpha_3 + \theta_2) = 0 \\ \Phi_{12} &\equiv x_6 + R_{56} \sin \theta_6 - x_5 - R_{65} \sin \theta_5 = 0 \\ \Phi_{13} &\equiv y_6 - R_{56} \cos \theta_6 - y_5 + R_{65} \cos \theta_5 = 0 \\ \Phi_{14} &\equiv x_7 - R_{67} \sin(\beta_1 - \theta_7) - x_6 - R_{76} \sin \theta_6 = 0 \\ \Phi_{15} &\equiv y_7 - R_{67} \cos(\beta_1 - \theta_7) - y_6 + R_{76} \cos \theta_6 = 0 \\ \Phi_{16} &\equiv x_8 + R_{87} \sin(\beta_2 + \theta_7) - x_7 + R_{78} \sin \theta_8 = 0 \\ \Phi_{17} &\equiv y_8 - R_{87} \cos(\beta_2 + \theta_7) - y_7 - R_{78} \cos \theta_8 = 0 \\ \Phi_{18} &\equiv x_8 - R_{18} \sin \theta_8 - x_1 - R_{81}^x = 0 \\ \Phi_{19} &\equiv y_8 + R_{18} \cos \theta_8 - y_1 + R_{81}^y = 0 \\ \Phi_{20} &\equiv x_6 - R_{16} \sin \theta_6 - x_1 - R_{61}^x = 0 \\ \Phi_{21} &\equiv y_6 + R_{16} \cos \theta_6 - y_1 + R_{61}^y = 0 \end{aligned}$$

Translational Constraints:

$$\begin{aligned} \Phi_{22} &\equiv y_1 - R_{4y} - y_4 = 0 \\ \Phi_{23} &\equiv \theta_1 - \theta_4 = 0 \end{aligned}$$

Driving Constraint:

$$\Phi_{24} \equiv x_1 - f(t) = 0 \quad (\text{A.10})$$

The Eqs. (A.10) are differentiated twice w.r.t. time to obtain  $\mathbf{C}_q \in \mathbf{R}^{24 \times 24}$ ,  $\mathbf{Q}_c \in \mathbf{R}^{24 \times 1}$ . Also we know  $\mathbf{M} \in \mathbf{R}^{24 \times 24}$ ,  $\ddot{\mathbf{q}} \in \mathbf{R}^{24 \times 1}$ ,  $\mathbf{Q}_e \in \mathbf{R}^{24 \times 1}$  and  $\mathbf{Q}_v \in \mathbf{R}^{24 \times 1}$ . Hence we have a set of 48 algebraic equations for the 48 unknowns i.e. the 24 system coordinates  $q_k$  ( $k=1, \dots, 8$ ) and 24 Lagrange Multipliers  $\lambda_j$  ( $j=1, \dots, 24$ ) which can be evaluated in MATLAB.

## Acknowledgements

The authors would like to acknowledge the support of team members. The authors are also thankful to Director, CMERI, Durgapur, for his kind permission to publish this paper.

## References

- [1] Ryan, R. ADAMS - Multibody Systems Analysis Software, *Multibody Systems*, Handboomer Schielen (Editor), 1990.
- [2] Blundell M. V., "Full Vehicle Modelling and Simulation using the ADAMS Software System," *Autotech '91*, IMechE, 1991.
- [3] M. V. Blundell, B. Ozdalyan, "Anti-Lock Braking System Simulation and Modeling in ADAMS", *IEEE International Conference on Simulation*, 457, pp. 140-144., 1998.
- [4] Dynamic torque analysis of Ground Articulating Pipeline System for oil sands haulage, Frimpong S., *SCSC'04*, pp. 526-531.
- [5] Erdman, A. G., Sandor, G. N., and Kota, S., 2001, *Mechanism Design*, Fourth edition Prentice Hall
- [6] S. Tickoo, D. Maini, and V. Raina, *CATIA V5R16 for Engineers & Designers*, Dreamtech Press, 2007.
- [7] <http://www.mscsoftware.com/>
- [8] Zhe Li, Sridhar Kota, "Virtual Prototyping and Motion Simulation with ADAMS," *Journal of Computing and Information Science in Engineering*, Transactions of the ASME, Vol. 1, September 2001, pp. 276-279.
- [9] Greenwood, D. T., *Principles of Dynamics*, Second edition, Prentice Hall, 1998.

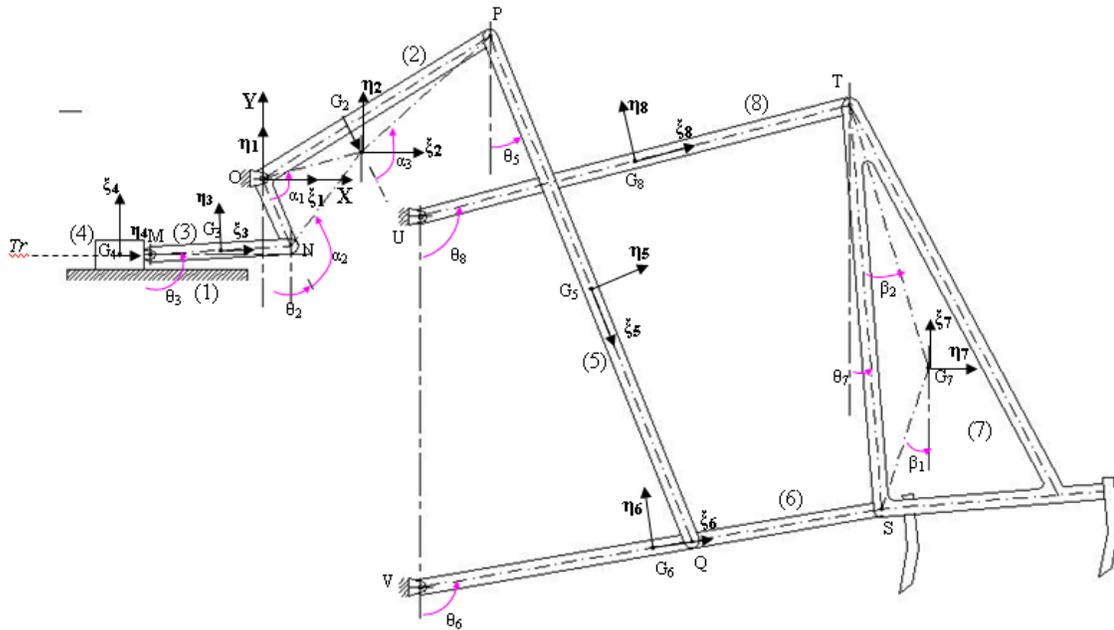


Fig. 6: Planar model of the tractor implement

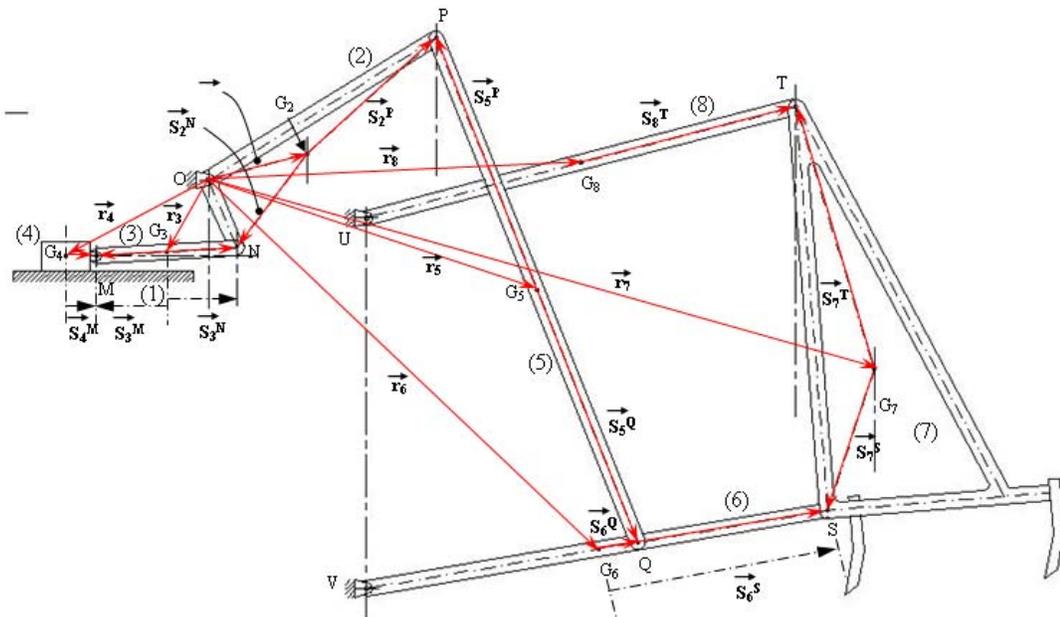


Fig. 7: Vector loops connecting different links of the tractor implement